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DISTRIBUTION OF RESOURCES IN A COMPETITIVE ENVIRONMENT

by

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Distribution of Resources in a Competitive Environment

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Abstract

When two agents of unequal strength compete, the stronger one is expected to always win the competition. This expectation is based on the assumption that evaluation of performance is flawless. If, however, the agents are evaluated on the basis of only a small sample of their performance, the weaker agent still stands a chance of winning occasionally. A theoretical analysis indicates that for this to happen, the weaker agent must introduce variability into the effort he or she invests in the behavior, such that on some occasions the weaker agent's level of performance is as high as that of the stronger agent, whereas on others it is null. This, in turn, would drive the stronger agent to introduce variability into his or her behavior. We model this situation in a game, present its game-theoretic solution, and report an experiment, involving 144 individuals, in which we tested whether players are actually sensitive to their relative strengths and know how to allocate their resources given those relative strengths. Our results indicate that they do.

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Distribution of Resources in a Competitive Environment

The world is full of competition: Flowers compete for pollination, animals compete for mating, and humans further compete for social standing. The processes by which such competitions are resolved are not foolproof, but rather are error prone. The uncertainty characteristic of the resolution of competition stems from the variability inherent in organisms' behavior, on the one hand, and the fallibility of the evaluation process, on the other. This fallibility is the result of limitations on the part of the agent who resolves the competition. For example, agents are often limited in the size of the sample they can draw from the distribution of competitors' behavior. Truly, a more able competitor is more likely to win the competition, but a weaker competitor is not doomed to always lose. This fact sustains diversity and maintains competition. Moreover, in the frequent and important case in which the resolving agent makes a choice between adaptive agents, who are eager to be selected (e.g., service providers), the very fallibility of the evaluation and selection process protects the resolving agent from being left at the mercy of a monopoly.

We have shown before (Avrahami, Güth, Kareev & Uske, 2007; Kareev & Avrahami, 2007) that when the level of effort is under the competitors' control, uncertainty in the evaluation process leads competitors to invest more. Here we ask how competitors whose resources are strictly limited behave when facing limited scrutiny. In particular, we ask whether in such circumstances, the weaker agents are sensitive to the relative strength of their competitors and allocate their own strength so as to maintain some chance of winning.

To put this question differently, one could ask what would be a "wise" distribution of resources when two competitors know each other's relative strength, but do not know in what field or on which occasion their performance may be checked. It seems reasonable that in this case, the weaker competitor should give up on some of the fields of competition, that is, not invest any resources in them, in order to be able to match the stronger competitor on the remaining ones. This allocation of resources means that if the competition happens to involve the former fields, the weaker competitor is bound to lose; but if the competition happens to involve the latter fields, he or she stands an equal chance of winning. The question still remains as to how many fields the weaker competitor should give up on, and how he or she should distribute resources in the remaining fields. Moreover, given that the weaker player is likely to introduce variability into his or her distribution of resources, the question of how best to distribute resources applies to the stronger competitor as well.

To answer these questions, we have construed the situation as a game of competition between two players of unequal strength. In the game, the two players are assigned different amounts of resources that they have to distribute, privately, in a fixed number of fields. Two fields, one for each player, are then drawn at random, and the player having more resources in his or her selected field wins. The game-theoretic solution (to be found in Hart, in press), is as follows: Assuming that the resources available to the two competitors are a and b, with $a \geq b > 0$, the optimal strategy for the stronger player is to divide his or her resources in a uniform distribution ranging from zero to twice that player's average resources among all fields on which evaluation may take place. The weaker player ought to give up on $1-\frac{b}{a}$ of the fields and distribute his or her resources in the remaining ones, following a pattern of allocation similar to that of the stronger player.[1]

The derivation of this solution is quite complex (see Hart, in press). Our main question is whether players with little mathematical sophistication reach the solution intuitively. A secondary question concerns the effect of practice on players' strategies. Do their strategies change over time? And if so, do their strategies change toward or away from the game-theoretic solution?

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[1] This is the solution when the quantities are continuous. See Hart (in press) for details of the more complex, but similar solution for the case in which quantities are discrete.

To find out, we tested 144 participants, matched in 72 pairs, who played the game just described over eight rounds. In most pairs, the two members differed in strength, but we assigned equal strength to some of the pairs, as a control. We found that, indeed, players' strategies were consistent with the game-theoretic optimal solution of the game.

Method

In the game, two players distributed 24, 18, or 12 stones among eight boxes. All six pair-wise combinations of number of stones were employed, producing three conditions of unequal strength (24-12, 18-12, and 24-18) and three conditions of equal strength (24-24, 18-18, and 12-12). Each pair of players participated in only one of these conditions. Once the players had completed their distribution, a die was thrown to determine which of each player's boxes would be opened. The numbers of stones in the opened boxes were then compared, and the player whose box contained a higher number of stones received a prize; the players split the prize in case of a draw.

Materials and Design

Eight opaque film boxes, numbered "1" to "8", were used for each player. The stones, of a type used for mosaics, were of two colors—one color for each player. An eight-sided die was used to determine which of the eight boxes would be opened. For the conditions of unequal strength, a coin was tossed to determine which of the two players would be assigned the larger number of stones. As already noted, there were three conditions in which the players differed in the number of stones they received (24-12, 18-12, and 24-18). Players thus differed in their strength to a large degree (2:1 ratio), to a medium degree (3:2 ratio), and to a small degree (4:3 ratio). There were also three control conditions in which both players had the same number of stones (24-24, 18-18, and 12-12). The control conditions were included so we could disentangle the effect of number of stones available to a player from effects, if found, of the ratio between the number of stones a player had and the number of stones the player's opponent had.

Procedure

After the game was described to a pair of players, a coin was tossed to determine who would be player A and who would be player B. Whenever the players differed in strength player A was the stronger of the two. Stones were then provided accordingly. Players privately distributed all their stones in their boxes as they saw fit, covered the boxes, and waited for the roll of the die. The die was rolled twice, once for each player. Each player then opened the box whose number matched the outcome of the die; the number of stones in the two boxes was compared, and the winner received a chip to be converted to money at the end of the experiment. Every pair played the game eight times. A chip was worth 5 NIS (a little over $1) when the players were of equal strength. When the players were of unequal strength, a chip was worth more to the weaker player than to the stronger player, to help compensate for the difference in the chances of winning and thus avoid having the weaker player feel that the game was unfair. Specifically, the chip was worth 4 NIS (player A) and 8 NIS (player B) when player B had half the strength of player A, 4 NIS (player A) and 6 NIS (player B) when player B had two thirds the strength of player A, and 4.5 NIS (player A) and 6 NIS (player B) when player B had three fourths the strength of player A (see Table 1).

Participants

One hundred forty-four students at the Hebrew University, Mount Scopus campus, participated in the experiment: 96 (48 pairs) in the three asymmetric conditions and 48 (24 pairs) in the three control conditions. There were thus 16 pairs in each of the unequal conditions and 8 pairs in each of the equal, control conditions.

Results

Correspondence to the Game-Theoretic Solution

The focus of our analysis of performance was the mean number of boxes left empty by the players in each condition. This information is presented in the right-most columns of Tables 1a (for the unequal pairs) and 1b (for the equal pairs). It is clear that when the players had unequal strength, the number of boxes left empty by participants in the role player A did not change much as the relative strength of the opponent diminished, whereas participants in the role of player B left more boxes empty as their strength relative to that of their competitor diminished. This pattern of results is fully in line with the game-theoretic solution. An analysis of variance of these data, with pair as the unit of analysis, player (A or B) and round (1–8) as within-pairs variables, and ratio as a between-pairs variable revealed, in addition to a strong main effect of player ($F[1,45]=51.88$, $p<.001$), an interaction between player and ratio ($F[2,45]=3.53$, $p=.038$).

Number of stones			Reward for winning (NIS)		Boxes left empty	
Player A	Player B	Ratio: A/B	Player A	Player B	Player A	Player B
24	12	2/1	4	8	0.453	3.188
18	12	3/2	4	6	1.094	2.422
24	18	4/3	4.5	6	1.070	2.398

a.

Number of stones			Reward for winning (NIS)		Boxes left empty	
Player A	Player B	Ratio: A/B	Player A	Player B	Player A	Player B
24	24	1/1	5	5	1.234	
18	18	1/1	5	5	0.984	
12	12	1/1	5	5	0.984	

b.

Table 1. a) Unequal pairs (16 in each condition); b) Equal pairs (8 in each condition)

To test whether the strategy of participants in the role of player B was related to the relative strength of the opponent, and not simply the result of having fewer stones to distribute, we conducted three additional analyses. These analyses focused on single players who had the same number of stones but faced opponents of different strengths. One analysis involved all players with 12 stones, another involved all players with 18 stones, and a third involved all players with 24 stones. Table 1 shows that the mean number of empty boxes for players with 12 stones was 0.984 when the opponent had 12 stones, 2.422 when the opponent had 18 stones, and 3.188 when the opponent had 24 stones. The effect of ratio (i.e., the relative strength of the opponent) was significant ($F[2,37]=7.160$, $p=.002$, p for the linear contrast =.001). The means for players with 18 stones facing opponents with 12, 18, or 24 stones were 1.094, 0.984, and 2.398, respectively. As predicted, the means were similar for players who faced a weaker opponent and players who faced an equally strong opponent, but the mean increased for players who faced a stronger opponent. The effect of the strength of the opponent was still significant, because of the difference between the first two means and the third ($F[2,37]=6.463$, $p=.004$). No effect of opponent was expected for players with 24 stones, who always faced either equal or weaker opponents. Indeed, the means for these players were 0.453, 1.070, and 1.234 when they faced opponents with 12, 18, or 24 stones, respectively; though these means increased slightly as the strength of the opponent increased, they did not differ significantly ($F[2,37]=1.959$, $p=.155$, p for the linear contrast =.100).

Another way to appreciate the correspondence between players' behavior and the game-theoretic predictions is to consider the actual wins in the game. When opponents are of equal strength, each is expected to win half of the time. When they differ in strength, the chance of the stronger one winning increases rapidly with the difference in strength, as the chance of the weaker player winning is only half the number of bins not left empty. Specifically, assuming again that the stronger player has *a* stones and the weaker player has *b* stones, and that they play according to the game-theoretic solution, the stronger one is expected to win in $1-\frac{b}{2a}$ of the cases. Table 2 presents the proportion of expected and actual wins of player A in the three unequal-strength conditions.

Number of Stones (Player A - Player B)	Expected Wins	Actual Wins
24 - 12	.75	.73
18 - 12	.67	.70
24 - 18	.63	.61

Table 2: Proportion of expected wins and proportion of actual wins of player A in the three conditions of unequal strength.

The table shows a very close correspondence between expected and actual wins. A comparison of the actual and expected values revealed that they did not differ (F<1).

Change Over Time

To answer our second question, concerning the effect of practice, we inspected the play of only the pairs with unequal strength. Figure 1 presents the number of empty boxes by round, separately for players A and B. As the figure shows, the difference between the players increased over time: Player A left slightly fewer boxes empty as time went on, whereas Player B left more. This indicates that the players' behavior better approximated the mathematical solution over time. The interaction between player (A or B) and round (1–8) was significant ($F[7,315]=3.97$, $p<.001$); more important, the linear-linear component of this interaction was significant ($F[1,45]=11.685$, $p=.001$). A planned analysis of the effect of round, separately for each player in the unequal-strength conditions, revealed a significant linear contrast for player B ($F[1,45]=8.100$, $p=.007$), but not for player A ($F[1,45]=3.234$, $p=.079$).

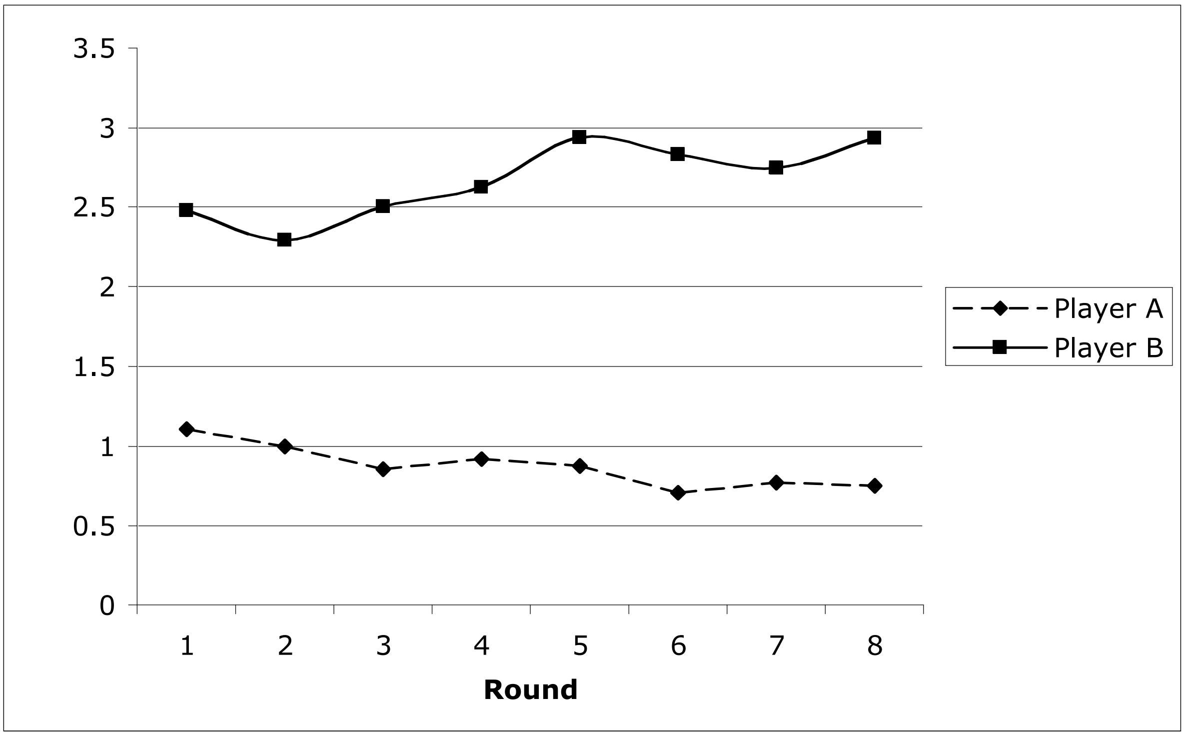

Figure 1. The number of boxes left empty by players of unequal strength over the eight rounds of the game, separately for players A and B.

General Discussion

Resource are limited: A day has only 24 hours, the amount of money in a person's bank account is finite, one's physical strength is limited, and the span of attention is bounded. Limitations in resources are of particular importance because daily interactions often involve competition, with the outcomes determined by the strength one can exhibit. It is with limited resources that people compete for standing in society. Does this mean that weaker competitors are doomed always to lose? Or do the weaker competitors stand a chance of winning—at least some of the time?

If competition took place at all times and involved all aspects of behavior, the true total of one's available resources would be revealed, and the weaker agent would always lose. Given, however, that competition takes place only some of the time and involves only some aspects of behavior, a wise distribution of resources can give even a weaker competitor a chance of winning. Once the weaker player adopts a "wise" strategy, one that calls for the introduction of variability into the distribution of resources, the stronger competitor must also introduce some variability into his or her distribution, to still guarantee a greater chance of winning than the weaker competitor has. Note that this analysis implies that, in those few fields in which the weaker agent chooses to compete, the distribution of that person's investments will match that of the stronger agent.

This analysis may explain why spending on clothing or on weddings is often similar in poor and in middle-class families, or at least more similar than the financial resources available to the two classes would lead one to expect. It may also explain why people who invest in intellectual improvement differ in the variety of topics they pursue and allocate their time to. The weaker ones usually limit the range of their interests, aiming at depth in only a few chosen topics, whereas the stronger ones attempt to excel in a greater variety of topics.

Although these phenomena are common and highly familiar, they cannot serve as proof that the theoretical analysis presented here is correct, because alternative explanations may be offered. The results of the experiment reported here, in which naïve players behaved in a way that approximated the sophisticated game-theoretic solution, indicates that the game models familiar situations. Thus, it may indeed be people's awareness of the scarcity of competition-resolution occasions and their sensitivity to their competitors' relative strength that promotes the behaviors observed above.

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