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Science, Beliefs and Knowledge: A Personal Reflection on Robert J. Aumann's Approach

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Robert Aumann, the 2005 Nobel Prize Laureate in Economics, has contributed some of the most original and important ideas in modern game theory and theoretical economics. Aumann has had an immense influence on these fields through his own works and also through his many collaborations and students. In the 1990's, I had numerous discussions and exchanged hundreds of e-mails with Aumann on various scientific and political matters, including the "Bible Code" controversy. This paper begins with a personal reflection on some of Aumman's scientific work and his views on the foundations of game theory, economics and science, and goes on to discuss Aumann's approach to the "Bible Code" issue, and the more general question: "How should scientists deal with extremely improbable claims?"

For a thorough discussion of Aumann's mathematical and scientific work and its impact the reader is referred to the chapter by Hart and Neyman (1995) [1]. To give the reader some feeling for his contributions let me mention two famous results which are both related to notions of equilibria. Aumann is famous for introducing games with a continuum of players and economies with a continuum of traders as models for studying large numbers of inter-

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[1]Sergiu Hart and Abraham Neyman (eds.), Game and Economic Theory, The University of Michigan Press, 1995.

acting agents. The basic question of understanding economies with a large number of traders goes back a long way. When all the traders are relatively small (in their market share) several "miracles" occur, and this situation is referred to in the economic literature as "perfect equilibrium theory". The first conjecture in this direction was made by Edgeworth in the late nineteenth century concerning the behavior of large economies was only solved for some models in the early 1960s. (The first theorem was due to Herbert Scarf and Gerard Debreu for replica economies.) The model of an economy with a continuum of players very clearly exhibits Edgeworth's phenomenon, as well as others which have not been proven for any other model. In an early seminal work (also from the early Sixties), Aumann proved Edgeworth's conjecture for the model of an economy with a continuum of traders. One of the milestones in this line of research is the book by Aumann and Lloyd Shapley on values of non-atomic games and, in particular, their theorem which identifies the non-atomic game's value and competitive equilibrium. The notion of a continuum of players is a technical-mathematical machinery which, like a new type of lens, enables us to see phenomena that could not otherwise be seen sharply, if at all.

Aumann's notion of correlated equilibrium is a brilliant extension of John Nash's famous notion of equilibrium for non-cooperative games. Correlated equilibrium points are Nash equilibrium points with mixed strategies that can depend on correlated random signals received by the players. Once the notion of mixed strategies (the interpretation of which is a foundational issue in game theory that many, including Aumann, have studied) is accepted, the notion of correlated equilibrium has advantages in terms of interpretation as well as computational complexity. The notion of correlated equilibrium provides a link between the classical Bayesian approach and the equilibrium concept.

Aumann has made other profound contributions concerning the equilibria of games and, in particular, those of repeated games with incomplete information. Indeed, the Economics 2005 Nobel Prize citation[2] emphasizes

[2]See "Conflicts and Cooperations through the Lens of Game Theory", (short version: http://nobelprize.org/economics/laureates/2005/press.html; long version:

Aumann's works on repeated games, some of which were written in collaboration with Michael Maschler. The original draft of this paper (written in 2002) did not mention repeated games but the following anecdote may give a taste of what they involve. A couple of years ago, Aumann told me (again) the story of how he decided to move to Jerusalem, just hours after saying yes to an offer from Bell Laboratories. I asked him if he now thought his choice to come here was the right one. Aumann seemed very surprised by my question and said: " Surely you understand, Gil, that it is much too early to tell". In a nutshell, in a repeated game with an "infinite horizon" it is always much too early to tell.

To continue on the personal note allow me to mention two papers of Aumann's which are not necessarily among his best-known but which particularly impressed me. They perhaps demonstrate the bold nature of what he regards as the horizons of this discipline.

As an undergraduate student I had a sort of a private seminar with a fellow student named Ariel Rubinstein (today a famous economist). We tried to read some papers in game theory together. I think we went through two or three papers, one of which was by Aumann and Mordecai Kurz on taxation and power. What impressed me in Aumann and Kurz's model was the combination of economic and political considerations and its far-reaching consequences. While game theory has had considerable impact on both economic thinking and political science, models which combine them are rare; yet this combination seems crucial to the understanding of economic phenomena which are hard to explain from a "pure" economic perspective (unemployment, for example).

In a different paper by Aumann and Michael Maschler a certain puzzling ruling in the Talmud is studied. The ruling concerns the division of a man's estate among his three widows (essentially this is a bankruptcy problem). The paper identifies the Talmud's solution as a game theoretic solution called the nucleolus and shows how this concept is derived from the basic rules of the Talmud (found in a different chapter) on how to divide disputed assets. Aumann and Maschler were influenced by Barry O'Neill's earlier work on

http://nobelprize.org/economics/laureates/2005/info.pdf

game theory and the Talmud. This particular talmudic question was proposed by Shlomo Aumann, Robert Aumann's son, who was a law student and at the same time a student in a yeshiva (a Jewish institute of higher learning). (Shlomo Aumann was called to active reserve duty in the 1982 war in Lebanon and was killed in action.)

Why is this paper interesting? The first goal of the paper was, of course, to understand the ruling of the Talmud. The initial observation that the Talmud's ruling coincides with a certain basic game-theoretic solution was remarkable but unsatisfactory. The mathematical definition of the game-theoretic concept seemed very remote from any type of reasoning used by the Talmud sages. Aumann and Maschler went on to explain how the solution could be derived from basic talmudic rules. This is a more appealing explanation of the ruling which does not rely on any sophisticated mathematics and at the same time provides an elegant axiomatic interpretation of the nucleolus (or kernel) for a certain class of games. Aumann and Maschler's finding is in line with Aumann's general view of game theory which I shall presently describe. Aumann has also always been fascinated by foundational questions of law and especially by its relation to game theory. Law is an ideal context in which to examine game theoretic ideas and to implement game-theoretic designs.

But what is the meaning of games, equilibrium points and mixed strategies? These questions were addressed by Aumann who has made many contributions to the foundations of game theory and is interested in foundational questions in economics, statistics and science as a whole. He has studied the interpretation of mixed strategies and Nash equilibrium points and the notion of subjective probabilities, as well as foundational questions concerning utilities, information, knowledge, altruism, evolution and consciousness. Aumann is especially famous for his works on knowledge and common knowledge. An agent's knowledge in an economic situation is, of course, crucial to his actions, and the agent's knowledge includes his knowledge of other agents' knowledge and his knowledge of the other agents' knowledge of his own knowledge, and so on. Aumann wrote an influential one-page paper asserting that under a certain assumption of complete knowledge, "agreeing

to disagree" is not possible.

Aumann's philosophy of economic theory and science as a whole is bold and far-reaching. Good places to read about this approach are van Damme (1998) and Aumann (2001).[3] In Aumann's view the basic role of game theory and theoretical economics, as well as other sciences, is to offer understanding [4], and (as in other sciences) that understanding should lead to verifiable (and falsifiable) predictions, to normative solutions and to engineering. One of Aumann's favorite examples[5] of the interplay between understanding, predictions, empiricism and engineering is the Gale-Shapley theoretic solution to the problem of assigning students to colleges (commonly known as the Gale-Shapley stable marriage theorem and algorithm). The Gale-Shapley result was followed by: the discovery by Al Roth that the same algorithm evolved (before the Gale-Shapley paper) in the U. S. and other places for the purpose of assigning medical interns to hospitals; the subsequent work by Roth and coauthors on expert labor markets; Roth's related laboratory experiments, and finally, Roth's "engineering" of an algorithm for markets with "two-body" problems. Aumann was also enthusiastic about the recent verification of the von Neumann-Morgenstern min-max mixed strategies in the observed serving strategies of tennis players at Wimbledon. In sum, Aumann expects theoretical economics and game theory to offer normative solutions to cases of strategic conflict and to offer insights and even predictions on observed economic behavior. Furthermore, he believes that one of the major challenges to game theorists and theoretical economists is to apply their knowledge to engineering.

There are, of course, other approaches. For example, Ariel Rubinstein, who is also very much interested in foundational issues, does not feel that predicting or understanding the behavior of economic agents or providing normative "solutions" is the goal of game theory and theoretical economics. Rather,

[3] "On the state of the art of game theory, an interview with Robert Aumann" by E. van Damme, *Games and Economic Behavior*, 24(1998) 181-210 and, R. J. Aumann, "Game Theory, Bilbao 2000", *Games and Economic Behavior* 45 (2003), 2-14.

[4] In Aumann's view, "seeking understanding" is a more profound characterization of science than "seeking the truth".

[5] Skeptics claim that this is the *only* example of its kind.

Rubinstein regards game theory and theoretical economics as tools "to establish 'linkage' between the concepts and statements that appear in our daily thinking on economic situations" (see Rubinstein (1998, 2000, 1995)).[6] Rubinstein's ideal view of academic scholarship is quite close to the nature of study in a yeshiva. I regard each of Aumann's and Rubinstein's approaches as romantic[7] in its own way.

Let me now move on to the issue of the "Bible Code". Professor Eliyahu Rips, a senior mathematician in my department, along with Doron Witztum, developed a methodology for extracting certain hidden messages in the Hebrew text of the Bible and statistically testing the hypothesis of a hidden code in the Bible. A paper by Witztum, Rips and Rosenberg in *Statistical Science* in 1994[8] reported a successful experiment conducted by the authors. Professor Rips is no doubt a wonderful mathematician and his surprising contributions to group theory had a huge impact on this field as well as on low-dimensional topology. Nevertheless, his research on the Bible Codes was received with skepticism by most scientists and even with some contempt. Rips was certainly not the first scientist to try to combine science and theology or even to extract hidden predictions of the future from the Bible. Newton invested a great deal of effort in trying to extract concrete predictions of the future from the Bible. (Incidently, many of Newton's writings on this matter were donated to the Hebrew University and are located in our National Library on the Givat Ram campus.) There is less tolerance today than there was in Newton's time for scientists attempting to use science in order to prove and apply religious doctrines. One primary reason is the weakened role of religion in today's society and another is our deeper understanding of the limitations of science itself.

[6]See the last chapter of: A. Rubinstein, *Modeling Bounded Rationality*, MIT Press, 1998 or the introduction to: A. Rubinstein, *Economics and Language*, Cambridge University Press, 2000. My favorite source on Rubinstein's approach is the following article in Hebrew: http://www.princeton.edu/~ariel/articles/academy.pdf.

[7]By romantic I mean, roughly, that it appeals to our emotions, sentiments, and inner-soul experiences.

[8]Doron Witztum, Eliyahu Rips, and Yoav Rosenberg, "Equidistant Letter Sequences in the Book of Genesis", *Statistical Science*, Vol. 9 (1994) 429-438.

How should we react to very unlikely or even absurd scientific claims? We had better ignore most, if not all of them. However, if we choose to react, what is the most appropriate way of doing so? Concerning the question of whether we should relate to unlikely claims at all, let me mention a mathematician who is one of the great number theorists of the twentieth century who has spent a considerable amount of time reading and finding the mistakes in proposed proofs (usually by laymen) for Fermat's last theorem. I often wonder why he has taken the trouble all these years. I think the primary reason is his sense of responsibility as a scientist and perhaps also his sympathy towards people who share his dreams, if not his abilities. He perhaps also realizes that laymen occasionally propose useful mathematical ideas and, although it is a remote possibility, a layman may arrive at a valid proof for Fermat's last theorem. (I don't think he would have spent the same amount of time on proposals for trisecting an arbitrary angle into three equal parts using a compass and a ruler, which has been proven impossible.)

For many years Aumann thought that an ironclad case had been made for the "Codes" and perhaps the first question that comes to mind is: "How could Aumann have been so gullible?" This is a good reminder that in this respect, scientists are not immune to lapses in judgements and perhaps may be even more prone to them than others. Aumann was also clearly a novice, and maybe even naive with respect to (practical) empirical science, and at times his respect for the Bible Code's proponents (and perhaps even his attitude to the Bible itself) clouded his judgement. But the story does not end there. Aumann put a lot of effort into studying the matter and even, to some extent, put his reputation on the line. Aumann's approach to investigating the issue was slow going and led to some confusion, but overall it was a powerful one.

Quite possibly, the biggest difference between my view on the Bible Codes and that of Aumann was in our priors. I regard the whole thing as absurd (similar to the search for a way to trisect an angle with a compass and ruler[9])

[9]Or to claims made for highly diluted homeopathic medications, or that uncomputable functions (in the technical sense) can be computed in the human brain, or that people can fly (literally) by practicing some sort of meditation.

while Aumann regarded it as extremely unlikely (perhaps as unlikely as a layman offering an elementary proof for Fermat's last theorem). Be that as it may, Aumann was convinced that Rips and Witztum's claim was a factual matter that could be decided by applying scientific methodology in a level-headed and objective way *without* taking priors into account. There is one system which routinely deals with claims that are often extremely unlikely, deceptive and even absurd, namely the judicial system. There are, of course, considerable differences in methodology but I think that Aumann's approach was influenced by that system.[10]

In 1996, Dror Bar-Natan tried carefully to reproduce the Bible Code experiment done by Witztum et al. He discovered a characteristic of the experiment that offered a simple alternative explanation for its success. Bar-Natan realized that of all the different names and "nicknames" of the 32 Rabbis included in the experiment — all of which he expected to be used by the experimenters — only about half were actually used.[11] Since the outcome of the experiment depends on the actual list of names used, Bar-

[10]Aumann was involved in several aspects of the work of Witztum, Rips and Rosenberg and its publication. At some point, Professor Perci Diaconis, a Stanford statistician, challenged the statistical test originally used and offered an alternative approach. (Diaconis' point was valid but this was not the crux of the matter.) When, in 1991, after several months of programming and computations, a new test was carried out and turned out successful, Aumann enthusiastically informed his close friend, Professor Benjy Weiss, who didn't quite share Aumann's enthusiasm. Weiss replied with the following story: A Rabbi in a small Jewish community in the U. S. used to tell his congregation about the merits of the Land of Israel, and the beauty of Jerusalem in his weekly sermons. When he reached the age of seventy his congregation decided to give him the gift of a trip to Israel, which he had never visited. After returning from his trip, the Rabbi started his usual weekly sermon: "You may remember all the lies I told you about the Land of Israel and the beauty of Jerusalem for all these years", he said. Then he paused for a moment, sighed and continued: "well, they are all completely true."

[11]Here, I assume some knowledge of the Bible Code experiment done by Witztum et al.. Very briefly, their experiment involves a list of 32 rabbis and a longer list of "appellations" (names and "nicknames") used for these rabbis in the rabbinic literature. (The same rabbi often appears in the rabbinic literature under various appellations.) The paper by Witztum et al. attributed the list of appellations they were using to an independent expert.

Natan's finding suggested that the success reported in the paper by Witztum, Rips and Rosenberg, reflected the choice of the rabbis' names — which names were accepted and which names were rejected. The freedom to choose the various names for the rabbis on the list was unknown to most of the people involved in the experiment and its evaluation, including Rips and Aumann. Bar-Natan's crucial observation was central to the 1999 paper by Brendan McKay, Dror Bar-Natan, Maya Bar-Hillel and myself in *Statistical Science*[12] which presented a comprehensive critical study of the paper by Witztum, Rips and Rosenberg and rejected their conclusions. This paper and a few related works gave ample evidence that the success of the experiment by Witztum et als. and that of several other experiments that were examined, simply reflected the tuning of the data to achieve the desired outcomes.[13] Our paper also reported the results of several independent "Bible Code" experiments conducted by McKay, Professor Barry Simon and others that had all failed. In other words, they had found no traces of the alleged "Bible Code".[14]

Shortly after Bar-Natan's observation and in light of other critiques of the Witztum et al. experiment, Aumann came to the conclusion that the best way to proceed was to conduct a careful, controlled and well-documented experiment. In 1996 Aumann formed a five-member committee that included Rips and Bar-Natan. The committee had around 20 meetings, during which the original experiments were carefully studied and new experiments carefully planned. The committee decided to conduct two experiments based on data selected by several experts. These two experiments attempted to repli-

[12]B. McKay, D. Bar-Natan, M. Bar-Hillel and G. Kalai, "Solving the Bible Code Puzzle", *Statistical Science*, Vol 14(1999) 150-173.

[13]E. Kass the former Executive Editor of *Statistical Science*, wrote in his introduction to the paper by McKay et als.: "Because minor variations in data definitions and the procedure used by Witztum et al. produce much less striking results, there is good reason to think that the particular forms of words those authors chose effectively 'tuned' their method to their data, thus invalidating their statistical test."

[14]The reader can find our paper and links to eight papers (Internet publications) by Doron Witztum, which attack our methodology, at McKay's Internet site http://cs.anu.edu.au/~bdm/dilugim/torah.html. Witztum's site and his recent book also contain a description of a large number of new successful experiments.

cate a successful experiment reported by Howard Gans. All the meetings of the committee were taped and transcripts were made.[15] The wording of the formal report was agreed upon by the committee in advance and only the p-values (the measures of statistical success) had to be filled in. The formal report[16] also contains an interpretation of the results according to the resulting p-values. Both experiments conducted by the committee failed — no traces for the "Bible Code" were found.

For anyone who had viewed the "Bible Code" as a serious scientific possibility, the conclusion of Aumman's committee was a serious setback. This was especially so because the committee included the leading Bible Code proponent, as well as some scientists who, prior to the Committee's work, did not dismiss the idea out of hand. [17]

In the popular view of science, scientists are, objective and reliable agents who are committed to a common scientific endeavor with a shared basic scientific picture. The ideal Bayesian view is sharply different. It permits scientists to be eccentric human agents who work on the edge of what is known. These agents are sometimes biased and often lack objectivity. However, the methodology of proofs and refutations, experiments and replications will eventually separate out the nonsense from what will become solid scientific knowledge and it will overcome scientists' tendency of self-deception

[15]The cost of the experts' work and the transcription service was paid for in equal parts by "Esh Hatorah", an "outreach" Jewish organization which uses the Bible codes in its "Discovery" seminars, and by Mr. Alec Gindis who was always a strong opponent of the codes.

[16]DP 364, Center for the Study of Rationality, Hebrew University, 2003.

[17]Aumann's own analysis of the outcomes, which also concluded his overall interest in the matter, is given in DP 365, Center for the Study of Rationality, Hebrew University, 2003. This DP also contains critique of the Committee's experiments by one of its members and by Witztum (which also reflects Professor Rips' opinion). Aumann's concluding point is: "We come finally to the bottom line: A priori the thesis of the codes research seems wildly improbable. Though the original work of Witztum, Rips and Rosenberg, and that of Gans, established a prima facie case for the existence of the codes, this case was undermined by the work of the "opponents". Research conducted under my own supervision failed to confirm the existence of the codes — though it did not established their non-existence. So I must return to my a priori estimate that the Codes phenomenon is improbable."

and even occasional cases of intentional deception. Scientific methodology is the invisible hand which allows the imperfect input of individual scientists to slowly aggregate into our joint common scientific knowledge. Aumann, subscribes to this view, but, unlike many scientists, he regards it not only as a philosophy of science, but also as a practical road-map for scientific activity.

Robert Aumann is an eminent scientist. He is also an observant Jew and religion has an important place in his life. He can be extremely funny and induce a lot of joy and he can be a formidable opponent in a good academic fight. His actions and views reflect strong convictions and deep thought as well as inner-conflicts. Arguing intensely with Aumann on the "Bible Code" (and various other matters) was both interesting, frustrating and a lot of fun[18] —he is often painfully provocative and skeptical on matters that others are not (or the other way around). At the end of the day, Robert Aumann is a champion of the scientific endeavor.

[18]Once we discovered that on a certain matter we agree! This was in the mid 90's and we both thought, contrary to the conventional view, that the State of Israel should welcome a free open democratic elections in the Palestinian Authority in which all groups (including the Hamas movement) can participate. We were so surprised to find that we shared the same opinion, that we decided to write a "letter to the editor" containing our position on the matter. When we got to the fine details, "agreeing to agree" was impossible.