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TOURNAMENTS WITH MIDTERM REVIEWS

by

ALEX GERSHKOV and MOTTY PERRY

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CENTER FOR THE STUDY
OF RATIONALITY

Feldman Building, Givat-Ram, 91904 Jerusalem, Israel
PHONE: [972]-2-6584135 FAX: [972]-2-6513681
E-MAIL: ratio@math.huji.ac.il
URL: http://www.ratio.huji.ac.il/

Tournaments with Midterm Reviews*

Alex Gershkov
Department of Economics,
University of Bonn
Alex.Gershkov@uni-bonn.de

Motty Perry
Department of Economics,
Hebrew University of Jerusalem
and The University of Chicago
Motty@huji.ac.il

Abstract

In many tournaments investments are made over time and conducting a review only once at the end, or also at points midway through, is a strategic decision of the tournament designer. If the latter is chosen, then a rule according to which the results of the different reviews are aggregated into a ranking must also be determined. This paper takes a first step in the direction of answering how such rules should be optimally designed.

A characterization of the optimal aggregation rule is provided for a two-agent two-stage tournament. In particular, we show that treating the two reviews symmetrically may result in an equilibrium effort level that is inferior to the one in which only a final review is conducted. However, treating the two reviews lexicographically by first looking at the final review, and then using the midterm review only as a tie-breaking rule, strictly dominates the option of conducting a final review only. The optimal mechanism falls somewhere in between these two extreme mechanisms. It is shown that the more effective the first-stage effort is in determining the final review's outcome, the smaller is the weight that should be assigned to the midterm review in determining the agents' ranking.

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1 Introduction

Lazear and Rosen (1981) were the first to notice that in many circumstances it is optimal to set up compensation on the basis of rank order, and that certain puzzling features of markets are easily explained in these terms. The vast economic literature that followed adopted their model in assuming that tournaments are like "all-pay auctions" in which agents choose their effort levels simultaneously at the start, and then prizes are allocated according to the resulting ranking. A very partial list includes the papers of Green and Stokey (1983), Dixit (1987), Krishna and Morgan (1998), Nalebuff and Stiglitz (1983), and Moldovanu and Sela (2001), to name just a few. In many tournaments however, investments are made over time and whether to conduct a review only once at the end, as most tournament models do, or also at some points midway through, is a strategic decision of the mechanism designer. Moreover, once such an option is considered, the first question that comes to mind is, how are the results of the different reviews optimally aggregated into a ranking? This is the question we seek to address in this paper.

Midway reviews are a common phenomenon in tournaments. Students compete to be ranked high in their class, and the professor must choose whether to conduct a final exam only, or final *and* midterm exams. If the latter is chosen, then a rule according to which the results of the two exams are aggregated into a ranking must also be determined. Assistant professors compete for tenure slots over a period of a few years. In some universities a midterm review is conducted and letters of reference are sent midway. Employees exert effort in order to be promoted in organizational hierarchies. Periodical reviews are conducted and these reviews are then aggregated to determine who will be promoted. The rounds system employed in many branches of sports is an obvious example of determining the winner by aggregating the results of a few rounds.

In this paper we study a simple two-stage two-agent tournament in which agents first choose their effort level in stage one, and then the level of effort in stage two. A designer whose goal is to maximize the agents' total effort has to decide whether to conduct only one final review after the second stage, or two reviews: a midterm and a final. In the model studied here, the review process is not perfect and can yield only an ordinal ranking that is positively, but only partially, correlated with the agents' efforts. In particular, we assume that the probability of the review resulting in agent i being ahead of agent j is increasing in the difference between their respective efforts, and the probability of an inconclusive review, where a tie is announced, is maximized when both agents exert the same effort level. While effort level invested in stage two affects only the final review, the effort invested in stage one might affect the outcomes of both reviews. This is captured by assuming that a discounted value of effort exerted in stage one enters into the process according to which the outcome of the final review is determined (where the discounting parameters can be anything between zero and one). Finally, we assume that the outcome of the midterm review, if conducted, is public knowledge.[1]

As expected, conducting a midterm review has two opposite effects: it tends to increase the agents' effort level in stage one, but tends to decrease it in stage two. The latter occurs when the midterm review results in one agent being ahead of the other. While we show that it is always strictly optimal to conduct a midterm review, we also demonstrate that this is true *only when* the results of both reviews are aggregated optimally. In particular, we show that treating the two reviews symmetrically might result in an equilibrium effort level that is inferior to the one in which only a final review is conducted. However, treating the two reviews lexicographically by first looking at the final review, and using the midterm review only as a tie-breaking rule, strictly dominates the option of conducting final review only. The optimal mechanism falls somewhere in between these two extreme mechanisms, as our characterization will show. In particular, we shall show that the more effective the first-stage effort is in determining the final review's outcome, the smaller is the weight that should be assigned to the midterm review in determining the agents' ranking.

Related Literature

Rosen (1986) and Gradstein and Konrad (1999), among others, studied a different version of multi-round tournament called the *Elimination Tournament.* In the elimination tournament the agents are divided into groups and only the winner of each round proceeds to the next round, in which he competes against the winners of other groups. The goal is to design an optimal structure of prizes at every round and an optimal assignment of contestants into groups.

A paper that is closer to ours is Aoyagi (2004), who studied a multi–stage two-agent tournament. However' unlike the case studied here, Aoyagi assumes a fixed mechanism, one in which equal weights are assigned to all midway reviews. In an environment in which first-stage effort is as effective as second-stage effort, relative to the final outcome, Aoyagi asked and provided an answer to when is it optimal to reveal to the participants information about the outcome of the midterm review. Yildirim (2005) also studied a two-stage two-agent contest in which agents observe each other effort in stage one before investing in stage two. However, in this model there is only one review at the end. Yildirim analyzes the effect of the asymmetric abilities on the equilibrium strategies of the players.

Dubey and Haimanko (2003) studied the effect of aggregating the results of reviews on the incentives of the contestants. They assume that the principal samples a number of rounds and the winner is the agent who wins the most rounds (among the sampled ones). They show that as the number of rounds goes to infinity the proportion of stages that are sampled goes to zero. This result is driven by sufficient differences in the contestants' quality.

The remainder of the paper is organized as follows. Section 2 describes the basic setup. In Section 3 we study the equilibrium when the designer is restricted to conduct a final review

[1] Aoyagi (2004) considers an environment in which revealing the review's result is a strategic choice of the designer.

only. The equilibrium when midterm and final reviews are conducted is analyzed in Section 4, and the optimal aggregation rule is then characterized in Section 5

2 Basic Setup

Two risk-neutral agents $i = 1, 2$ are asked to exert effort in two stages. Agent $i's$ effort in stage $t = 1, 2$ is denoted by $e_i^t \in [0, \infty)$ and is exerted at cost $c : [0, \infty) \to R_+$. Effort e_i^t is agent i's private information and is not observed by either the other agent or the principal. The principal, however, whose goal is to maximize the expected sum of effort $\sum_i \sum_t e_i^t$, can influence the agents' decision by conducting reviews and rewarding the agents in a way that reflects the reviews' results. Reviews can take place either after stage one (hereafter the midterm review), after stage two (the final review), or after both stages, and we assume throughout the paper that the reviews' results are public information.

We restrict our attention to the case in which there is a fixed prize of size one that has to be allocated at the end of stage two. The prize might be promotion to a higher rank in the corporation, or it might be thought of as the utility of a student from being ranked first in his class. Thus, if agent i whose effort levels in the two stages are e_i^1 and e_i^2, respectively, wins the award in probability p, his expected payoff is $p - \sum_t c(e_i^t)$.

The review process is imprecise and yields only a noisy ranking of agents' efforts. In particular, the outcome of a midterm review, if conducted, is determined by

$$\Gamma\left(e_1^1, e_2^1\right) = \left[f_1\left(e_1^1 - e_2^1\right),\ \ f_0\left(e_1^1 - e_2^1\right),\ \ f_2\left(e_1^1 - e_2^1\right)\right]$$

where $f_1(\cdot)$ is the probability that agent 1 comes out first in the review. Similarly $f_2(\cdot)$ is the probability that the second agent is shown to exert more effort, and $f_0(\cdot) = 1 - f_1(\cdot) - f_2(\cdot)$ is the probability that the review is inconclusive and a tie is declared.

Let $\tau_i = \delta e_i^1 + e_i^2$ denote the effective total effort where $\delta \in [0, 1]$ captures the fact that the effort in stage one is not as effective as the effort in stage two in determining the outcome of the final review, which is similarly determined by

$$\Gamma\left(e_1^1, e_2^1, e_1^2, e_2^2\right) = \left[f_1\left(\tau_1 - \tau_2\right),\ \ f_0\left(\tau_1 - \tau_2\right),\ \ f_2\left(\tau_1 - \tau_2\right)\right].$$

We make the following assumption on $f(\cdot)$ and $c(\cdot)$.

As: Symmetry: for all $y \in (-\infty, \infty)$,

$$f_0(y) = f_0(-y)\,,\ \ \textit{and}\ \ f_1(y) = f_2(-y)\,.$$

Ai: Information content

a. $\frac{df_1(x)}{dx} > 0$; $\frac{df_2(x)}{dx} < 0$; $\frac{df_0(x)}{dx} \begin{cases} > 0 \;\; if \;\; x < 0 \\ = 0 \;\; if \;\; x = 0 \\ < 0 \;\; if \;\; x > 0 \end{cases}$,

b. $\lim_{x\to-\infty} f_1(x) = 0$ and $\lim_{x\to\infty} f_1(x) = 1$.

At: Γ is twice continuously differentiable.

Ac: $c(0) = 0$, $c'(0) = 0$ and for any $e \in [0, \infty]$, $c''(e) > \gamma(\Gamma) > 0$[2].

Note that **As** expresses the symmetry between the two agents, while **Ai** captures the idea that the review process is informative. More precisely, the probability of coming out first (second) increases (decreases) in one's own effort, while the probability of a tie is maximized when both agents choose the same effort level. Assumptions **At** and **Ac** are mainly technical and much more than what is needed to assure that second-order conditions for optimum are met and that a symmetric equilibrium exists. In particular, the cost function $c(\cdot)$ must be convex enough, so that its second derivative is always above some constant γ that in turn depends on the shape of Γ.

The figure below illustrates the main features of $\Gamma(\cdot)$. In particular, note that $f_1(0) = f_2(0) < 1/2$ and that $f_0(\cdot)$ reaches its maximum at 0.

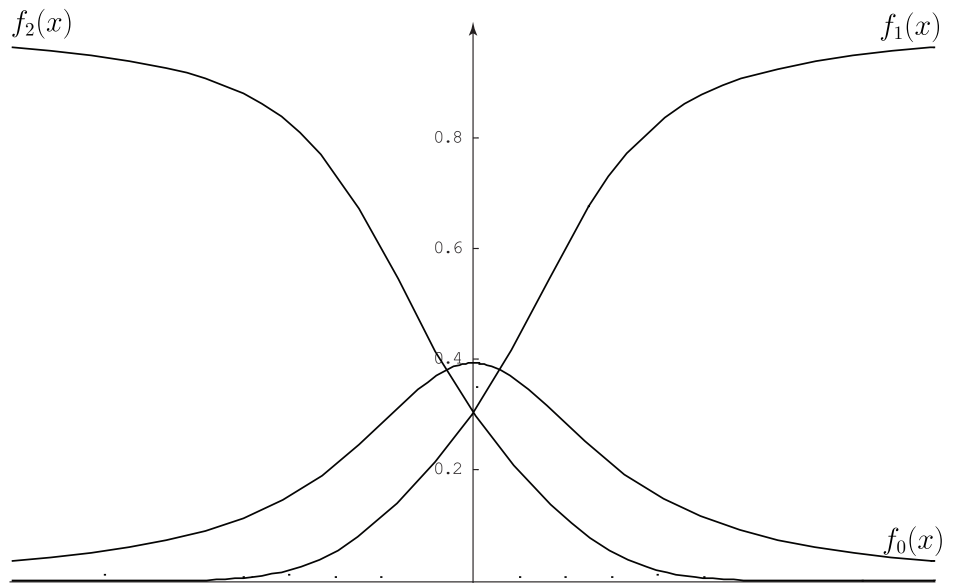

Figure 1: The main features of $\Gamma(\cdot)$.

Denote the output of the $t's$ review by $s_t \in \{0, 1, 2\}$. A review system, often called a *mechanism*, specifies how many reviews to conduct, when to conduct them, and how the different s_t are then to be aggregated to yield an allocation rule. We restrict our attention to symmetric mechanisms. In the following section we start by studying the mechanism where only one review is conducted.

[2] The precise form of $\gamma(\Gamma)$ is defined in Appendix B.

3 Conducting Final Review Only

When only a final review is conducted, the set of symmetric mechanisms is characterized by the probability $\beta \in [0,1]$ at which the prize is allocated to the agent who came out first in the review. Thus, the expected utility of agent i whose effort levels in the two stages are e_i^1 and e_i^2, and his opponent's effort levels are e_j^1 and e_j^2, and as defined above $\tau_l = \delta e_l^1 + e_l^2$ for $l = i, j$, is

$$-c(e_i^1) - c(e_i^2) + \beta f_i(\tau_1 - \tau_2) + \frac{1}{2} f_0(\tau_1 - \tau_2) + (1-\beta) f_j(\tau_1 - \tau_2) \tag{1}$$

which by **As** can be written as

$$-c(e_i^1) - c(e_i^2) + \beta f_1(\tau_i - \tau_j) + \frac{1}{2} f_0(\tau_i - \tau_j) + (1-\beta) f_2(\tau_i - \tau_j).$$

The two first-order conditions with respect to e_i^1 and e_i^2 are given by

$$c'(e_i^1) = \delta\left[\beta f_1'(\tau_i - \tau_j) + \frac{1}{2} f_0'(\tau_i - \tau_j) + (1-\beta) f_2'(\tau_i - \tau_j)\right] \tag{2}$$

and

$$c'(e_i^2) = \left[\beta f_1'(\tau_i - \tau_j) + \frac{1}{2} f_0'(\tau_i - \tau_j) + (1-\beta) f_2'(\tau_i - \tau_j)\right] \tag{3}$$

First note that for $\beta \leq \frac{1}{2}$ the best response of each agent is to choose zero effort level in every stage. Also recall from Assumption **Ai** that $f_0'(0) = 0$. Therefore, for any $\beta > \frac{1}{2}$, there exists a symmetric solution to (2) and (3) where $e_1^1 = e_2^1 = \hat{e}^1$ and $e_1^2 = e_2^2 = \hat{e}^2$ for which

$$\begin{aligned} c'(\hat{e}^1) &= \delta(2\beta - 1) f_1'(0) \\ c'(\hat{e}^2) &= (2\beta - 1) f_1'(0) \end{aligned} \tag{4}$$

¿From the assumed convexity of $c(\cdot)$ and the monotonicity of f_1 it follows that a designer whose goal is to maximize the agents' efforts will set $\beta = 1$ which allows us to rewrite (4) as

$$\begin{aligned} c'(\hat{e}^1) &= \delta f_1'(0) \\ c'(\hat{e}^2) &= f_1'(0). \end{aligned} \tag{5}$$

In the appendix we provide conditions under which second-order conditions for maximization are also satisfied. Thus, the solutions to (5) determines the effort level in the symmetric equilibrium.

Remark: *A second look at (4) reveals that the rule according to which the prize is allocated following a tie has no effect on incentives. In particular, choosing to allocate the prize in some probability $\alpha \in [0,1)$ when the review is inconclusive will not change the effort level in equilibrium. This is a consequence of the fact that when both agents choose the same effort level, then Assumption **Ai** implies that a small* *change in the effort level of some agent in either stage will not change the probability of a tie.*

*Indeed, this result can be derived also from Assumption **As**. To see why, assume that after a tie the prize is allocated to player i with probability α_i and denote by Δ_Z the change in the probability of outcome $Z \in \{0,1,2\}$ when player 1 increases his effective effort from τ_1 to $\tau_1 + \varepsilon$. That is,*

$$\Delta_Z = f_Z(\tau_1 + \varepsilon - \tau_2) - f_Z(\tau_1 - \tau_2).$$

When the agent considers changing the effective effort from τ_1 to $\tau_1 + \varepsilon$ he compares the effect of this change on his cost, and the effect on the probabilities of the different outcomes, which is

$$\beta\Delta_1 + \alpha_1\Delta_0 + (1-\beta)\Delta_2.$$

Because $\sum_Z f_Z(x) = 1$ we must have

$$\Delta_1 + \Delta_0 + \Delta_2 = 0.$$

*Finally, if $\tau_1 = \tau_2$, then for any ε, **As** implies that $\Delta_1 = -\Delta_2$. Therefore, $\Delta_0 = 0$, which in turn implies that the allocation rule after a tie has no effect on the agents' incentives to exert effort.*

Remark: *In our setup δ captures elements not under the principal's control and elements under his control. For example, δ might capture the fact that time spent early on studying toward the final is less effective than time spent just before the final, which of course is not under the principal's control. On the other hand, by shifting more of the final weight to materials that are covered early on, the principal can make the time invested early on more effective. Similarly, in a promotion decision in a corporation, the committee in charge of promotion might be more affected by the latest achievements of the different candidates. The designer might instruct the committee how to treat the candidates' achievements in both periods however, this recommendation may have only a partial effect. In light of this discussion, it is instructive to note that total effort level $\hat{e}^1 + e^2$ increases with the discount factor δ.*

A corollary of this observation is that conducting only a final review dominates a mechanism in which only a midterm review is conducted. To see why, simply note that when only a midterm review is conducted, second-stage effort has no effect on the allocation of the prize, and hence the effort levels in equilibrium $\bar{e}^1$ and $\bar{e}^2$ are exactly the mirror image of the effort levels when only a final review is conducted and $\delta = 0$.

4 Conducting Midterm and Final Reviews

Recall that $s_t \in \{0,1,2\}$ stands for the result of review $t \in \{1,2\}$ and let $g(s_1, s_2)$ be the probability that the prize goes to agent 1 after the history (s_1, s_2). Because we are restricting our attention to the symmetric mechanism it follows that

$$g(s_1, s_2) = 1 - g(s'_1, s'_2) \text{ where } s'_i = \begin{cases} s_i & if \quad s_i = 0 \\ 2 & if \quad s_i = 1 \\ 1 & if \quad s_i = 2 \end{cases}.$$

Our interest lies in characterizing the optimal values of $g(s_1, s_2)$. At the end of stage one a midterm review is conducted and results either in a tie or in a winner. Now, although the effort level of agent j in stage one is not revealed to agent i, in equilibrium agent i knows its value. Thus, by abusing the language somewhat, we refer to the different continuations following the midterm as subgames. With this in mind, let $e_i^L(e_i^1)$ be agent $i's$ optimal effort level in the subgame when he is the leader, after exerting e_i^1 in stage one. Similarly, let $u_i^L(e_i^1, e_i^L(e_i^1))$ denote his expected utility in the subgame. Along the same line define $e_i^F(e_i^1),\ u_i^F(e_i^1, e_i^F(e_i^1))$ for the subgame in which he is a follower and $e_i^T(e_i^1)$, $u_1^T(e_1^1, e_1^T(e_1^1))$ for the subgame following a tie.

Agent $i's$ expected utility in the mechanism can now be written as

$$\begin{aligned} &-c(e_i^1) + f_i\left(e_1^1 - e_1^1\right) u_i^L(e_i^1, e_i^L(e_i^1)) + f_0\left(e_1^1 - e_1^1\right) u_i^T(e_i^1, e_i^T(e_i^1)) \\ &+ f_j\left(e_1^1 - e_1^1\right) u_i^F(e_i^1, e_i^F(e_i^1)) \end{aligned}$$

Using **As**, it can be rewritten as

$$\begin{aligned} &-c(e_i^1) + f_1\left(e_i^1 - e_j^1\right) u_i^L(e_i^1, e_i^L(e_i^1)) + f_0\left(e_i^1 - e_j^1\right) u_i^T(e_i^1, e_i^T(e_i^1)) \\ &+ f_2\left(e_i^1 - e_j^1\right) u_i^F(e_i^1, e_i^F(e_i^1)) \end{aligned} \tag{6}$$

In equilibrium, e_i^1, $e_i^L(e_i^1)$, $e_i^F(e_i^1)$, and $e_i^T(e_i^1)$ maximize agent i's payoff given the strategy of his rival, agent j, e_j^1, $e_j^L(e_j^1)$, $e_j^F(e_j^1)$, and $e_j^T(e_j^1)$. The following lemma, which derives the first order condition of the agents' maximization problem, is instrumental in characterizing the optimal mechanism. The proof of it is given in the appendix.

Lemma 1. *The effort levels $\overline{e}^1$, e^T, and $e^L = e^F = e^{LF}$ are a solution to the first-order conditions if they satisfy the following three equations:*

$$c'(e^T) = \begin{cases} f_1'(0)\left(g(0,1) - g(0,2)\right) & if \quad g(0,1) - g(0,2) > 0 \\ 0 & otherwise \end{cases} \tag{7}$$

$$c'(e^{LF}) = \begin{cases} f_1'(0)\left(g(1,1) - g(1,2)\right) & if \quad g(1,1) - g(1,2) > 0 \\ 0 & otherwise \end{cases} \tag{8}$$

and

$$c'(\overline{e}^1) = \begin{cases} f_1'(0)\, A & if \quad A > 0 \\ 0 & otherwise \end{cases} \tag{9}$$

where

$$A = [2f_1(0)(g(1,1)+g(1,2)-1)+f_0(0)(2g(1,0)-1)]$$
$$+2f_1(0)\,\delta\,[g(1,1)-g(1,2)]+f_0(0)\,\delta\,[g(0,1)-g(0,2)]$$

Proof. See Appendix. □

Denote by $\overline{e}^1(g(s_1,s_2))$, $e^{LF}(g(s_1,s_2))$ and $e^T(g(s_1,s_2))$ the solutions to (9), (8) and (7). Assume for now that for every $g(s_1,s_2)$, the effort levels $\overline{e}^1(g(s_1,s_2))$, $e^{LF}(g(s_1,s_2))$, and $e^T(g(s_1,s_2))$ constitute a symmetric equilibrium. In other words, $\overline{e}^1(g(s_1,s_2))$, $e^{LF}(g(s_1,s_2))$, and $e^T(g(s_1,s_2))$ define a global maximum for each agent, given that the other agent is doing the same. In Section 5 we derive the optimal allocation rule, $g^*(s_1,s_2)$, and in the appendix we provide sufficient conditions on $c(\cdot)$ and Γ under which $\overline{e}^1(g^*(s_1,s_2))$, $e^{LF}(g^*(s_1,s_2))$, and $e^T(g^*(s_1,s_2))$ determine the symmetric equilibrium.

Remark: *Note that the agents' efforts in period two are not affected by how the prize is allocated after a tie in the final, as can be seen from the absence of the term $g(\cdot,0)$ in either (8) or (7). But unlike the case where only final review is conducted, here $g(\cdot,0)$ does have an effect on the agents' incentives, and in particular on the effort exerted in stage one as can be seen in 9. Consequently, when two reviews are conducted, the allocation rule after a tie in the final must be chosen with care. However, the allocation rule after ties in both reviews $g(0,0)$ has no effect on incentives (note that the term $g(0,0)$ does not appear in (8), (7) or 9.)*

5 The Optimal Allocation Rule

The optimal allocation rule $g^*(s_1,s_2)$ solves

$$g^*(s_1,s_2) = \arg\max_{g(s_1,s_2)}[\overline{e}^1(g(s_1,s_2))+2f_1(0)e^{LF}(g(s_1,s_2)) +(1-2f_1(0))e^T(g(s_1,s_2))]. \tag{10}$$

The following theorem follows immediately from the first-order conditions that were derived in Lemma 1, and the monotonicity of f_1 and c'.

Theorem 1. *In the optimal symmetric allocation rule $g^*(s_1,s_2)$,*

$$\begin{aligned} g^*(1,1) &= 1-g^*(2,2)=1 \\ g^*(1,0) &= 1-g^*(2,0)=1 \\ g^*(0,1) &= 1-g^*(0,2)=1 \\ g^*(0,0) &= 0.5 \end{aligned}$$

Proof. From (9) and (8) it follows that effort levels in stage one and in the subgame in which there is a leader increase with $g^*(1,1)$. Because effort level in the subgame in which there is a tie is not affected by $g^*(1,1)$, we conclude that in the optimal mechanism $g^*(1,1) = 1$. Similarly, $g^*(0,1) = 1$ follows because effort levels in stage one and in the subgame in which there is a tie increase with $g^*(0,1)$ (see (9) and (7)) while $g^*(0,1)$ does not have an effect on the effort level in the subgame in which there is a leader (see (8)). Finally, note that $g^*(1,0) = 1$ follows because effort level in stage one increases with $g^*(1,0)$ (see (9)), but has no effect on the effort level in stage two. $g^*(0,0) = 0.5$ follows from the symmetry of the mechanism. □

Theorem 1 characterizes all values of $g^*(s_1, s_2)$ but $g^*(1,2) = 1 - g^*(2,1)$, which turns out to depend on the specific parameters of the problem (like c and Γ) and will be studied shortly. But first note that all values of $g^*(s_1, s_2)$ that are determined by the Theorem agree with the simple and commonly used *majority rule* in which the two reviews are treated symmetrically. In the majority rule, an agent is awarded (say) two points after winning a review, one point after a tie, and zero otherwise, and the prize is awarded to the agent who collected more points in total. One might rush to conclude that a natural candidate for $g^*(1,2)$ is half, the value assigned to it by the majority rule. The following example demonstrates that choosing $g^*(1,2) = 0.5$ is often suboptimal and might even lead to an outcome that is inferior to the one obtained when only final review is conducted.

Example: Assume that $c(e) = \exp(e) - e$, $\delta = 1$, *and* $f_1(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\frac{(s-0.2)^2}{2})ds$. *It follows that in this case*

$$f_1'(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-0.2)^2}{2}\right)$$

and

$$\begin{aligned} f_1(0) &= 0.420\,74 \\ f_1'(0) &= 0.391\,04. \end{aligned}$$

Consider first the one-review system in which only a final review is conducted. In this mechanism, the first-order conditions yield $e_1 = e_2 = \bar{e}$ *where*

$$c'(\bar{e}) = f_1'(0).$$

It follows that in the one-review system, the effort level of each agent in each stage is $\bar{e} = 0.330\,05$.

Now consider the two-review system with the majority rule $g^m(s_1, s_2)$ *in which*

$$g^m(1,1) = g^m(1,0) = g^m(0,1) = 1 \quad \textit{and} \quad g^m(1,2) = g^m(0,0) = 0.5$$

is used. The solutions to the first-order conditions, $\bar{e}^1$, e^T and e^{LF}, are given by the system of equations

$$\begin{aligned} c'(e^T) &= f_1'(0) \\ c'(e^{LF}) &= \frac{1}{2}f_1'(0) \\ c'(\bar{e}^1) &= (1+\delta)f_1'(0)\left(f_1(0)+f_0(0)\right), \end{aligned}$$

which yields

$$\bar{e}^1 = 0.373\,65, \quad e^{LF} = 0.178\,58 \text{ and } e^T = 0.330\,05.$$

Thus, the expected effort level is 0.57624, *which is lower than* 0.6601, *the expected effort level when only a final review is conducted. It can be easily verified that the second-order conditions are satisfied as well.* ◁

The main objective of the example above was to demonstrate that unless the allocation rule is chosen carefully, one might end up with a mechanism that is inferior to the one in which only one review is conduct. While the exact mechanism, and in particular the exact value of $g^*(1,2)$, varies with the different parameters of the problem, the following lemma shows that a two-review system, in which the midterm review is used only as a tie-breaking device for the final review, i.e., $g(1,2)=0$, although not always optimal, dominates the one-review system.

Lemma 2. *If $g(1,2)=0$, then expected effort in the two-review system is higher than in the one-review system. That is,*

$$\bar{e}^1 + 2f_1(1)e^{LF} + (1-2f_1(1))e^T > \hat{e}^1 + \hat{e}^2.$$

Proof. Consider first the two-review system and note that when $g(1,2)=0$, then

$$e^T = e^{LF} = \bar{e}^2 \text{ where } c'(\bar{e}^2) = f_1'(0),$$

and the effort level in stage two is given by

$$c'(\bar{e}^1) = f_1'(0)\,[f_0(0)+\delta].$$

Next recall the equilibrium equation (5) for the one-review system, we conclude that (for all $f_0(0)>0$) $\bar{e}^2 = \hat{e}^2$ and $\bar{e}^1 > \hat{e}^1$. □

Note that the lower $g(1,2)$ is, the higher the incentives are to exert effort in the second stage. However, lowering $g(1,2)$ decreases the incentives of the agents to exert effort in the first stage because it decreases the weight assigned to a win in stage one. The optimal $g(1,2)$ exactly balances this trade-off. The following theorem characterizes $g^*(1,2)$ and in particular demonstrates that if $c''' > 0$ then $g^*(1,2)$ decreases as δ increases. In words, the more effective the first-stage effort is in determining the outcome of the final review, i.e., the higher δ is, the smaller is the weight that should be assigned to the midterm in determining the allocation of the prize. The intuition behind this result is rather straightforward. When δ is small the midterm review is the more effective tool to get the agents to exert effort in the first stage. But assigning a high weight to the midterm review has an adverse effect on the second-stage effort. Thus, when δ gets larger, first-stage effort has an effect on final review's outcome, and agents exert effort in stage one even when the weight that is assigned to the midterm review is very low. Because decreasing the weight assigned to the midterm review increases the expected effort in stage two, it is optimal to do so.

Theorem 2. *If $c'''(\cdot) > 0$, then*

$$g^*(1,2) = \begin{cases} \frac{2f_1(0)}{1+2f_1(0)} & \text{if } \delta = 0 \\ 0 & \text{if } \delta = 1 \end{cases}.$$

Moreover, $g^*(1,2)$ decreases with δ.

Proof. Recall that $g^*(1,2)$ is chosen to maximize

$$TE(g(1,2)) = \bar{e}^1(g(1,2)) + 2f_1(0)e^{LF}(g(1,2)) + (1 - 2f_1(0))e^T(g(1,2)).$$

It follows from (7) that $\frac{\partial e^T}{\partial g(1,2)} = 0$ and hence

$$\frac{\partial TE}{\partial g(1,2)} = \frac{\partial \bar{e}^1}{\partial g(1,2)} + 2f_1(0)\frac{\partial e^{LF}}{\partial g(1,2)}.$$

From (9)

$$\frac{\partial \bar{e}^1}{\partial g(1,2)} = -\frac{2f_1(0)\, f_1'(0)\,(1-\delta)}{-c''(e^1)} > 0$$

and from (8)

$$\frac{\partial e^{LF}}{\partial g(1,2)} = -\frac{-f_1'(0)}{-c''(e^{LF})}.$$

We get

$$\frac{\partial TE}{\partial g(1,2)} = \frac{\partial \bar{e}^1}{\partial g(1,2)} + 2f_1(0)\,\frac{\partial e^{LF}}{\partial g(1,2)} = 2f_1(0)\, f_1'(0)\left[\frac{1-\delta}{c''(e^1)} - \frac{1}{c''(e^{LF})}\right]. \tag{11}$$

First note that the assumed $c'''(\cdot) > 0$ guarantees that $\frac{\partial TE}{\partial g(1,2)} = 0$ is a point of maximum. Now, when $\delta = 0$, $\frac{\partial TE}{\partial g(1,2)} = 0$ implies that $\bar{e}^1 = e^{LF}$, which by plugging into (8) and (9) yields

$$g^*(1,2) = \frac{2f_1(0)}{1+2f_1(0)}.$$

Next note that from (9) it follows that $\bar{e}^1$ increases with δ and with $g(1,2)$ while e^{LF} decreases with $g(1,2)$. It is now easy to see from (11) that $g^*(1,2)$ decreases with δ. Finally, observe that when $\delta = 1$, $\frac{\partial TE}{\partial g(1,2)} < 0$ for any $g(1,2)$, and we conclude that in this case $g^*(1,2) = 0$. □

6 Appendix A.

Proof of Lemma 1. We are interested in showing that $\overline{e}^1(g(s_1,s_2))$, $e^{LF}(g(s_1,s_2))$, and $e^T(g(s_1,s_2))$ solve agent i's first-order condition system of equations, when agent j's effort levels are set to $\overline{e}^1(g(s_1,s_2))$, $e^{LF}(g(s_1,s_2))$ and $e^T(g(s_1,s_2))$. We start by deriving the FOC in the subgames following the midterm, assuming that in stage one agent i's choice is $\overline{e}^1(g(s_1,s_2))$. Consider the subgame following a tie. First note that although the midterm resulted in a tie, still the player that invested more in the first stage has an advantage since it increases the probability of winning the final review. Let $\tau^T(i,j) = \delta e_i^1 + e_i^T - \delta e_j^1 - e_j^T$, where e_j^T is the effort level of agent j in the subgame. Agent i's expected utility when his effort level is e_i^T is

$$-c(e_i^T) + f_1\left(\tau^T(i,j)\right) g(0,1) + f_0\left(\tau^T(i,j)\right) g(0,0) + f_2\left(\tau^T(i,j)\right) g(0,2). \tag{12}$$

Recall that $f_2(x) = 1 - f_1(x) - f_0(x)$, and rewrite (12) as

$$-c(e_i^T) + f_1(\tau^T(i,j))\,(g(0,1) - g(0,2)) + f_0(\tau^T(i,j))\,(g(0,0) - g(0,2)) + g(0,2).$$

Agent i's first-order condition is

$$c'(e_i^T) = \left[f_1'(\tau^T(i,j))\,(g(0,1) - g(0,2)) + f_0'(\tau^T(i,j))\,(g(0,0) - g(0,2))\right]. \tag{13}$$

Finally, note that because by assumption both agents exerted the same effort level in stage one then there exists a symmetric solution $e_1^T = e_2^T = e^T(g(s_1,s_2))$ that solves the first-order condition (13) for both agents for which

$$c'(e^T) = f_1'(0)\,(g(0,1) - g(0,2))$$

if $g(0,1) - g(0,2) > 0$, and $e^T = 0$ otherwise, as we had to show. Before moving to the subgame in which there is a leader, we note that the utility of agent i in this subgame is given by

$$\begin{aligned} u_i^T(e_i^1, e_i^T(e_i^1)) &= -c(e_i^T(e_i^1)) + f_1\left(\widetilde{\tau}^T(i,j)\right)(g(0,1) - g(0,2)) \\ &\quad + f_0\left(\widetilde{\tau}^T(i,j)\right)(g(0,0) - g(0,2)) + g(0,2) \end{aligned} \tag{14}$$

where $e_i^T(e_i^1)$ is a solution of the first-order condition, and

$$\widetilde{\tau}^T(i,j) = \delta e_i^1 + e_i^T(e_i^1) - \delta e_j^1 - e_j^T(e_j^1).$$

We next consider the subgame in which there is a leader. Assume agent i came out first in the midterm review and is now the leader. Using **As**, agent i's expected utility in the subgame when his effort level is e_i^L and his rival's effort level is e_j^F is

$$-c(e_i^L) + f_1(\tau^L(i,j))\,(g(1,1) - g(1,2)) + f_0(\tau^L(i,j))\,(g(1,0) - g(1,2)) + g(1,2)$$

where as before $\tau^L(i,j) = \delta e_i^1 + e_i^L - \delta e_j^1 - e_j^F$.

The leader's first-order condition is

$$c'(e_i^L) = f_1'(\tau^L(i,j))\,(g(1,1) - g(1,2)) + f_0'(\tau^L(i,j))\,(g(1,0) - g(1,2))\,. \tag{15}$$

Similarly, we can express the follower's expected utility and the corresponding first-order condition as

$$\begin{aligned}&-c(e_j^F) + f_1(-\tau^L(i,j))\,(g(1,1) - g(1,2)) + f_0(-\tau^L(i,j))(g(1,1) - g(1,0))\\&+(1 - g(1,1))\end{aligned}$$

and

$$c'(e_j^F) = f_1'(-\tau^L(i,j))\,(g(1,1) - g(1,2)) + f_0'(-\tau^L(i,j))(g(1,1) - g(1,0)). \tag{16}$$

As in the subgame following a tie, so here, because by assumption both players choose the same effort in stage one, there exists a symmetric solution $e_1^L = e_2^F = e^{LF}(g(s_1, s_2))$ which solves the two first-order conditions (15) and (16) and satisfies

$$c'(e^{LF}) = f_1'(0)\,(g(1,1) - g(1,2))$$

if $g(1,1) - g(1,2) > 0$ and $e^T = 0$ otherwise, as stated in the lemma.

It follows that the utilities of the agents in the subgame are given by

$$\begin{aligned}&u_i^L(e_i^1, e_i^L(e_i^1)) = -c(e_i^L(e_i^1)) + f_1\left(\widetilde{\tau}^L(i,j)\right)[g(1,1) - (g(1,2)]\\&+f_0\left(\widetilde{\tau}^L(i,j)\right)[g(1,0) - (g(1,2)] + g(1,2)\end{aligned} \tag{17}$$

and

$$\begin{aligned}&u_j^F(e_j^1, e_j^F(e_j^1)) = -c(e_j^F(e_j^1)) + f_1\left(-\widetilde{\tau}^L(i,j)\right)[g(1,1) - g(1,2)]\\&+f_0\left(-\widetilde{\tau}^L(i,j)\right)[g(1,1) - g(1,0)] + 1 - g(1,1)\end{aligned} \tag{18}$$

where $e_i^L(e_i^1)$ and $e_j^F(e_j^1)$ are the solutions to the system of two corresponding first-order conditions (15) and (16) and $\widetilde{\tau}^L(i,j) = \delta e_i^1 + e_i^L(e_i^1) - \delta e_j^1 - e_j^F(e_j^1)$.

Assuming now that agent i's effort levels in the subgames are $e^{LF}(g(s_1, s_2))$ and $e^T(g(s_1, s_2))$, it is left for us to show that

$$
\begin{aligned}
c'(\bar{e}^1) &= \\
&\quad f_1'(0)\left[2f_1(0)\left(g(1,1)+g(1,2)-1\right)+f_0(0)\left(2g(1,0)-1\right)\right] \\
&\quad +2f_1(0)\,\delta f_1'(0)\left[g(1,1)-g(1,2)\right] \\
&\quad +f_0(0)\,\delta f_1'(0)\left[g(0,1)-g(0,2)\right]
\end{aligned}
$$

if the expression on the right-hand side of the previous equality is positive. Recall that agent i's expected utility in the mechanism is

$$
\begin{aligned}
&-c(e_i^1)+f_1\left(e_i^1-e_j^1\right)u_i^L(e_i^1,e_i^L(e_i^1))+f_0\left(e_i^1-e_j^1\right)u_i^T(e_i^1,e_i^T(e_i^1)) \\
&+f_2\left(e_i^1-e_j^1\right)u_i^F(e_i^1,e_i^F(e_i^1))
\end{aligned}
$$

which yields the following first-order condition with respect to e_i^1 :

$$
\begin{aligned}
c'(e_i^1) &= f_1'\left(e_i^1-e_j^1\right)u_i^L(e_i^1,e_i^L(e_i^1))+f_1\left(e_i^1-e_j^1\right)\frac{du_i^L(e_i^1,e_i^L(e_i^1))}{de_1^1} \\
&\quad +f_0'\left(e_i^1-e_j^1\right)u_i^T(e_i^1,e_i^T(e_i^1))+f_0\left(e_i^1-e_j^1\right)\frac{du_i^T(e_i^1,e_i^T(e_i^1))}{de_1^1} \\
&\quad +f_2'\left(e_i^1-e_j^1\right)u_i^F(e_i^1,e_i^F(e_i^1))+f_2\left(e_i^1-e_j^1\right)\frac{du_i^F(e_i^1,e_i^F(e_i^1))}{de_1^1}.
\end{aligned} \tag{19}
$$

Using $f_2'(x)=-f_1'(x)-f_0'(x)$ we can rewrite (19) as

$$
\begin{aligned}
c'(e_i^1) = f_1'\left(e_i^1-e_j^1\right)\left[u_i^L(e_i^1,e_i^L(e_i^1))-u_i^F(e_i^1,e_i^F(e_i^1))\right] \\
+f_0'\left(e_i^1-e_j^1\right)\left[u_i^T(e_i^1,e_i^T(e_i^1))-u_i^F(e_i^1,e_i^F(e_i^1))\right]+f_1\left(e_i^1-e_j^1\right)\frac{du_i^L(e_i^1,e_i^L(e_i^1))}{de_1^1} \\
+f_0\left(e_i^1-e_j^1\right)\frac{du_i^T(e_i^1,e_i^T(e_i^1))}{de_1^1}+f_2\left(e_i^1-e_j^1\right)\frac{du_i^F(e_i^1,e_i^F(e_i^1))}{de_1^1}.
\end{aligned}
$$

Next note that for $Z\in\{T,L,F\}$ and for $i=1,2$,

$$
\frac{du_i^Z(e_i^1,e_i^Z(e_i^1))}{de_i^1}=\frac{\partial u_i^Z(e_i^1,e_i^Z(e_i^1))}{\partial e_i^1}+\frac{\partial u_i^Z(e_i^1,e_i^Z(e_i^1))}{\partial e_i^Z(e_i^1))}\frac{\partial e_i^Z(e_i^1))}{\partial e_i^1}.
$$

However, since $e_i^Z(e_i^1))$ maximizes $u_i^Z(e_i^1,e_i^Z(e_i^1))$ for any e_i^1, we have[3]

$$
\begin{aligned}
&\frac{du_i^L(e_i^1,e_i^L(e_i^1))}{de_i^1} \\
&= \delta\left[f_1'\left(\tilde{\tau}^L(i,j)\right)\left(g(1,1)-g(1,2)\right)+f_0'\left(\tilde{\tau}^L(i,j)\right)\left(g(1,0)-g(1,2)\right)\right],
\end{aligned}
$$

[3] Since $e_i^Z(e_i^1))$ maximizes $u_i^Z(e_i^1,e_i^Z(e_i^1))$, either $\frac{\partial u_i^Z(e_i^1,e_i^Z(e_i^1))}{\partial e_i^Z(e_i^1))}=0$ or $e_i^Z(e_i^1))=0$. Therefore, the second term in the last expression is 0.

$$\begin{aligned}
&\frac{du_i^F(e_i^1, e_i^F(e_i^1))}{de_i^1} \\
= \ &\delta\left[f_1'\left(-\widetilde{\tau}^L(j,i)\right)(g(2,1)-g(2,2)) + f_0'\left(-\widetilde{\tau}^L(j,i)\right)(g(2,0)-g(2,2))\right]
\end{aligned}$$

and

$$\begin{aligned}
&\frac{du_i^T(e_i^1, e_i^T(e_i^1))}{de_i^1} \\
= \ &\delta\left[f_1'\left(\widetilde{\tau}^T(i,j)\right)(g(0,1)-g(0,2)) + f_0'\left(\widetilde{\tau}^T(i,j)\right)(g(0,0)-g(0,2))\right].
\end{aligned}$$

Now, if in every second-stage subgame both agents exert the same effort (i.e., $e^{LF}(g(s_1, s_2))$ and $e^T(g(s_1, s_2))$), then both agents have the same first-order conditions determining first-stage effort. It implies that there exists a solution to the first-order condition (19) in which both agents choose the same effort in stage one. Hence, stage two's first-order conditions are indeed given by (7) and (8) and $\widetilde{\tau}^Z(i,j) = 0$ for $Z \in \{T, L, F\}$. Moreover, from (17) and (18) it follows that

$$\begin{aligned}
&u_i^L(e_i^1, e_i^L(e_i^1) - u_i^F(e_i^1, e_i^F(e_i^1)) \\
= \ &f_0(1)\left[2g(1,0) - g(1,2) - g(1,1)\right] - g(2,2) + g(1,2) \\
= \ &f_0(1)\left[2g(1,0) - 1\right] + 2f_1(1)\left[g(1,2) - 1 + g(1,1)\right].
\end{aligned}$$

Plugging the last expressions into (19) we get, using $f_1'(0) = -f_2'(0)$ and $f_0'(0) = 0$, the required equality (9), as stated in the lemma. ■

7 Appendix B. Second-Order Conditions

Lemma 3. *Assume there exists $\lambda > 0$, such that for any $x \in R$ the following holds:*

$$f_1'(x), f_0'(x), |f_0''(x)|, |f_1''(x)|, |f_2''(x)| < \lambda.$$

If for any $y \in R_+$

$$c''(y) > 5\delta^2\lambda + 5\delta\lambda^2 + 3\lambda$$

then in the one-review system as well as the two-review system, the solution to the first-order conditions solves the agent maximization problem. That is, the second-order conditions for maximization hold.

Before proving the statement of Lemma 3 we first show that increasing the effort level after a tie in the first stage always increases the agent's probability of winning the prize.

Claim 1: For any $x \in R$, $f_1'(x) + \frac{1}{2}f_0'(x) > 0$.

Proof: Observe first that for $x \leq 0$, the statement of the claim follows directly from **Ai**. Also note that **As** implies that

$$f_1(x) + \frac{1}{2}f_0(x) + f_1(-x) + \frac{1}{2}f_0(-x) = 1.$$

Differentiating with respect to x yields

$$f_1^{'}(x) + \frac{1}{2}f_0^{\prime}(x) = f_1^{'}(-x) + \frac{1}{2}f_0^{\prime}(-x)$$

which establishes the claim. ■

We first prove the statement for the one-review system in which only a final review is conducted.

Proof of Lemma 3 **for the one-review system.** Without loss of generality we restrict our attention to agent 1. Denote by $u_1(e^1, e^2)$ the expected utility of agent 1 in the second stage if his effort levels are e^1 and e^2, while his opponent plays $\overline{e}^1$,$\overline{e}^2$. Note that

$$\frac{\partial u_1(e^1, e^2)}{\partial e^2} = -c'(e^2) + \left[f_1' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) \right]. \tag{20}$$

Since $c'(0) = 0$, **Claim 1** implies that $c'(0) < f_1'(x) + \frac{1}{2}f_0'(x)$ for any $x \in R$. Moreover, $c'' > \frac{3}{2}\lambda \geq f_1^{'\prime}(x) + \frac{1}{2}f_0''(x)$ implies that for any e^1 there exists a unique positive solution to $\frac{\partial u_1(e^1,e^2)}{\partial e^2} = 0$ that maximizes $u_1(e^1, e^2)$, and denote this solution by $e^2(e^1)$. From the implicit function theorem it follows that

$$\frac{de^2(e^1)}{de^1} = -\delta \frac{f_1'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right)}{-c''(e^2) + f_1'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right)}.$$

Since $c''(e^2) > 3\lambda \geq 2\left(f_1'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right)\right)$, we can conclude that $\left|\frac{de^2(e^1)}{de^1}\right| < \delta$. Taking a derivative of agent 1's expected utility with respect to e^1 yields

$$-c'(e^1) + \frac{\partial u_1(e^1, e^2(e^1))}{\partial e^1} + \frac{\partial u_1(e^1, e^2(e^1))}{\partial e^2} \frac{de^2(e^1)}{de^1}.$$

The second derivative with respect to e^1 is given by

$$-c''(e^1) + \frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial \left(e^1\right)^2} + 2\frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial e^2 \partial e^1} \frac{de^2(e^1)}{de^1} + \frac{\partial u_1(e^1, e^2(e^1))}{\partial e^2} \frac{d^2 e^2(e^1)}{d\left(e^1\right)^2}$$
$$+ \frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial \left(e^2\right)^2} \left(\frac{de^2(e^1)}{de^1}\right)^2.$$

In what follows we will show that the above expression is negative. Since $e^2(e^1)$ maximizes $u_1(e^1, e^2)$, it is enough to show that

$$-c''(e^1) + \frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial \left(e^1\right)^2} + 2\frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial e^2 \partial e^1} \frac{de^2(e^1)}{de^1} < 0. \tag{21}$$

Starting with the second term in (21), it follows from $f_Z'' < \lambda$ for $Z \in \{0, 1\}$ that

$$\begin{aligned} & \frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial \left(e^1\right)^2} \\ = \; & \delta^2 \left(f_1'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) \right) \leq \frac{3}{2}\delta^2 \lambda. \end{aligned}$$

The third term in (21)

$$\begin{aligned} & \frac{\partial^2 u_1(e^1, e^2(e^1))}{\partial e^2 \partial e^1} \frac{de^2(e^1)}{de^1} \\ = \; & \delta \left(f_1'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) + \frac{1}{2} f_0'' \left(\delta e^1 + e^2 - \delta \overline{e}^1 - \overline{e}^2\right) \right) \frac{de^2(e^1)}{de^1} < \frac{3}{2}\delta^2 \lambda \end{aligned}$$

Since $c'' > 5\delta^2\lambda + 5\delta\lambda^2 + 3\lambda$, we conclude that the second-order condition is satisfied. ■

We are now ready to prove the statement of the lemma for the two-review system, in which both midterm and final reviews are conducted.

Proof of Lemma 3 for the two-review system. As before we restrict our attention to agent 1 only. Assume that the agent's opponent plays $\bar{e}^1$, $\bar{e}^T$ and $\bar{e}^{LF}$. Then

$$\begin{aligned} \frac{\partial u^L(e^1, e^L)}{\partial e^L} &= -c'(e^L) - f_2'(\delta e^1 + e^L - \delta \overline{e}^1 - \overline{e}^{LF})\left(1 - g(1,2)\right) \\ \frac{\partial u^T(e^1, e^T)}{\partial e^T} &= -c'(e^T) + f_1'(\delta e^1 + e^T - \delta \overline{e}^1 - \overline{e}^T) + \frac{1}{2} f_0'(\delta e^1 + e^T - \delta \overline{e}^1 - \overline{e}^T) \\ \frac{\partial u^F(e^1, e^F)}{\partial e^F} &= -c'(e^F) + f_1'(\delta e^1 + e^F - \delta \overline{e}^1 - \overline{e}^{LF})\left(1 - g(1,2)\right). \end{aligned} \tag{22}$$

Proceeding along the same lines as above, since $c'(0) = 0$, we get that $\frac{\partial u^Z(e^1, 0)}{\partial e^Z} > 0$ for any $Z \in \{L, F, T\}$. As before, $c'' > \frac{3}{2}\lambda \geq f_1''(x) + \frac{1}{2} f_0''(x)$ implies that for any e^1 there exists a unique positive solution to $\frac{\partial u^Z(e^1, e^Z)}{\partial e^Z} = 0$ that maximizes $u^Z(e^1, e^Z)$, which will be denoted by $e^Z(e^1)$. Similarly to the one-review system it can be shown that for any $Z \in \{L, F, T\}$

$$\left| \frac{de^Z(e^1)}{de^1} \right| < \delta.$$

Taking the derivative of agent 1's expected utility with respect to e^1 we obtain

$$\begin{aligned} & -c'(e^1) + f_1'\left(e^1 - \overline{e}^1\right) u^L(e^1, e^L(e^1)) + f_1\left(e^1 - \overline{e}^1\right) \frac{du^L(e^1, e^L(e^1))}{de^1} \\ & + f_0'\left(e^1 - \overline{e}^1\right) u^T(e^1, e^T(e^1)) + f_0\left(e^1 - \overline{e}^1\right) \frac{du^T(e^1, e^T(e^1))}{de^1} \\ & + f_2'\left(e^1 - \overline{e}^1\right) u^F(e^1, e^F(e^1)) + f_2\left(e^1 - \overline{e}^1\right) \frac{du^F(e^1, e^F(e^1))}{de^1} \end{aligned}$$

where

$$\frac{du^Z(e^1, e^Z(e^1))}{de^1} = \frac{\partial u^Z(e^1, e^Z(e^1))}{\partial e^1} + \frac{\partial u^Z(e^1, e^Z(e^1))}{\partial e_1^Z} \frac{de^Z(e^1)}{de^1}.$$

The second derivative is

$$
\begin{aligned}
& -c''(e^1) + f_1''\left(e^1 - \overline{e}^1\right) u^L(e^1, e^L(e^1)) + f_0''\left(e^1 - \overline{e}^1\right) u^T(e^1, e^T(e^1)) \\
& + f_2''\left(e^1 - \overline{e}^1\right) u^F(e^1, e^F(e^1)) + 2 f_1'\left(e^1 - \overline{e}^1\right) \frac{du^L(e^1, e^L(e^1))}{de^1} \\
& + 2 f_0'\left(e^1 - \overline{e}^1\right) \frac{du^T(e^1, e^T(e^1))}{de^1} + 2 f_2'\left(e^1 - \overline{e}^1\right) \frac{du^F(e^1, e^F(e^1))}{de^1} \\
& + f_1\left(e^1 - \overline{e}^1\right) \frac{d^2 u^L(e^1, e^L(e^1))}{d\left(e^1\right)^2} + f_0\left(e^1 - \overline{e}^1\right) \frac{d^2 u^T(e^1, e^T(e^1))}{d\left(e^1\right)^2} \\
& + f_2\left(e^1 - \overline{e}^1\right) \frac{d^2 u^F(e^1, e^F(e^1))}{d\left(e^1\right)^2}
\end{aligned} \tag{23}
$$

In what follows we will show that for any strategy ($\overline{e}^1$,$\overline{e}^T, \overline{e}^{LF}$) of agent two, the expression in (23) is always negative.

First note that

$$
\begin{aligned}
\frac{d^2 u^Z(e^1, e_1^Z(e^1))}{d\left(e^1\right)^2} &= \frac{\partial^2 u^Z(e^1, e_1^Z(e^1))}{\partial\left(e^1\right)^2} + 2\frac{\partial^2 u^Z(e^1, e_1^Z(e^1))}{\partial e^1 \partial e_1^Z} \frac{de_1^Z(e^1)}{de^1} \\
& + \frac{\partial u^Z(e^1, e_1^Z(e^1))}{\partial e_1^Z} \frac{d^2 e_1^Z(e^1)}{d\left(e^1\right)^2} + \frac{\partial^2 u^Z(e^1, e_1^Z(e^1))}{\partial\left(e_1^Z\right)^2} \left(\frac{de_1^Z(e^1)}{de^1}\right)^2.
\end{aligned}
$$

However, since for any e^1, e_1^Z maximizes $u^Z(e^1, e_1^Z(e^1))$, we have that

$$
\frac{d^2 u^Z(e^1, e_1^Z(e^1))}{d\left(e^1\right)^2} < \frac{\partial^2 u^Z(e^1, e_1^Z(e^1))}{\partial\left(e^1\right)^2} + 2\frac{\partial^2 u^Z(e^1, e_1^Z(e^1))}{\partial e^1 \partial e_1^Z} \frac{de_1^Z(e^1)}{de^1}. \tag{24}
$$

Next note that $u^Z(e^1, e^Z(e^1)) \in [0,1]$, for any $Z \in \{L, F, T\}$, because non-negative utility at any subgame is guaranteed by $e_1^Z = 0$ and since the prize is 1, the utility cannot be higher than 1. Therefore,

$$
\begin{aligned}
& f_1''\left(e^1 - \overline{e}^1\right) u^L(e^1, e^L(e^1)) + f_0''\left(e^1 - \overline{e}^1\right) u^T(e^1, e^T(e^1)) \\
& + f_2''\left(e^1 - \overline{e}^1\right) u^F(e^1, e^F(e^1)) \\
< \; & \max\left\{f_1''\left(e^1 - \overline{e}^1\right), 0\right\} + \max\left\{f_0''\left(e^1 - \overline{e}^1\right), 0\right\} + \max\left\{f_2''\left(e^1 - \overline{e}^1\right), 0\right\} \leq 3\lambda.
\end{aligned} \tag{25}
$$

Recall that

$$
\frac{du^Z(e^1, e^Z(e^1))}{de^1} = \frac{\partial u^Z(e^1, e^Z(e^1))}{\partial e^1} + \frac{\partial u^Z(e^1, e^Z(e^1))}{\partial e_1^Z} \frac{de^Z(e^1)}{de^1} = \frac{\partial u^Z(e^1, e^Z(e^1))}{\partial e^1}
$$

where the last equality follows from the envelope theorem. Therefore,

$$\begin{aligned}\frac{du^L(e^1,e^L(e^1))}{de^1} &= \delta\left(-f_2'\left(\delta e^1+e^L-\delta\overline{e}^1-\overline{e}^{LF}\right)\right)(1-g(1,2))\\ \frac{du^T(e^1,e^T(e^1))}{de^1} &= \delta\left[f_1'(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)+\frac{1}{2}f_0'(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)\right]\\ \frac{du^F(e^1,e^F(e^1))}{de^1} &= \delta\left[f_1'(\delta e^1+e^F-\delta\overline{e}^1-\overline{e}^{LF})(1-g(1,2))\right].\end{aligned}$$

Since $f_2'<0$, we can conclude that

$$\begin{aligned}& 2f_1'\left(e^1-\overline{e}^1\right)\frac{du^L(e^1,e^L(e^1))}{de^1}+2f_0'\left(e^1-\overline{e}^1\right)\frac{du^T(e^1,e^T(e^1))}{de^1}\\ & +2f_2'\left(e^1-\overline{e}^1\right)\frac{du^F(e^1,e^F(e^1))}{de^1}\\ \leq\ & 2\delta\lambda^2+3\delta\lambda^2=5\delta\lambda^2.\end{aligned} \tag{26}$$

From (24) it follows that

$$\begin{aligned}\frac{d^2u^L(e^1,e_1^L(e^1))}{d\left(e^1\right)^2} &< \left(-\delta^2-2\delta\frac{de_1^L(e^1)}{de^1}\right)f_2''\left(\delta e^1+e^L-\delta\overline{e}^1-\overline{e}^{LF}\right)(1-g(1,2))\\ &< 3\delta^2\left|f_2''\left(\delta e^1+e^L-\delta\overline{e}^1-\overline{e}^{LF}\right)\right|<3\delta^2\lambda.\end{aligned}$$

Similarly, we get that

$$\begin{aligned}\frac{d^2u^T(e^1,e_1^T(e^1))}{d\left(e^1\right)^2} &< \left(\delta^2+2\delta\frac{de_1^T(e^1)}{de^1}\right)\left(f_1''(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)+\frac{1}{2}f_0''(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)\right)\\ &< 3\delta^2\left(\left|f_1''(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)\right|+\frac{1}{2}\left|f_0''(\delta e^1+e^T-\delta\overline{e}^1-\overline{e}^T)\right|\right)<3\delta^2\frac{3}{2}\lambda\end{aligned}$$

and

$$\begin{aligned}\frac{d^2u^F(e^1,e_1^F(e^1))}{d\left(e^1\right)^2} &< \left(\delta^2+2\delta\frac{de_1^F(e^1)}{de^1}\right)(1-g(1,2))\,f_1''(\delta e^1+e^F-\delta\overline{e}^1-\overline{e}^{LF})\\ &< 3\delta^2\left|f_1''(\delta e^1+e^F-\delta\overline{e}^1-\overline{e}^{LF})\right|<3\delta^2\lambda.\end{aligned}$$

Therefore,

$$\begin{aligned}& f_1\left(e^1-\overline{e}^1\right)\frac{d^2u^L(e^1,e_1^L(e^1))}{d\left(e^1\right)^2}+f_0\left(e^1-\overline{e}^1\right)\frac{d^2u^T(e^1,e_1^T(e^1))}{d\left(e^1\right)^2}\\ & +f_2\left(e^1-\overline{e}^1\right)\frac{d^2u^F(e^1,e_1^F(e^1))}{d\left(e^1\right)^2}\\ \leq\ & 3\delta^2\frac{3}{2}\lambda.\end{aligned} \tag{27}$$

In sum, since the second line in (23) is less than 3λ, the third line is less than $5\delta\lambda^2$, and the last line is less than $3\delta^2\frac{3}{2}\lambda$, it follows that if

$$c'' > 5\delta^2\lambda + 5\delta\lambda^2 + 3\lambda,$$

then the second-order condition is satisfied. ■

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