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A THEORY OF OPTIMAL DEADLINES

by

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A THEORY OF OPTIMAL DEADLINES

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Abstract. This paper sets forth a model of contracting for delivery in an environment with time to build and adverse selection. The optimal contract is derived and characterized and it takes the form of a *deadline contract*. Such a contract stipulates a deadline for delivery for each possible type of agent efficiency. The optimal contract induces inefficient delay by using delivery time as a screening device. Furthermore, rents are decreasing in the agent's efficiency. In meeting the deadline, the agent's effort is strictly increasing over time, due to discounting. It is shown that increasing the project's gross value decreases delivery time, while the scale or difficulty of the project decreases it. Last, it is shown that the agent's rents are increasing in both project difficulty and gross project value.

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1. Introduction

A major consideration in scheduling any but the most standard project, is to estimate the duration of the development phase. In virtually all projects where completion takes time, such as weapons development, infrastructure, construction and software development, a procurer must determine the production schedule and in doing so, trade off different considerations such as speedy completion and the amount of committed resources. Since many such projects are executed to order by third parties, the question arises of how to provide correct incentives for the third party in order to align his interests with those of the procurer. Despite the issue's obvious practical importance, there has been surprisingly little attention paid in the economics literature to how such schedules are determined, and to how such schedules are influenced by incentive considerations.[1] For many such investment projects, timely completion is of paramount importance, as delays entail considerable foregone benefits (profits, or social benefits when the procurer is a government entity). To be sure, much attention has been given to the issue of forecasting development time.[2] Maybe not surprisingly, a lot of

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[1] For an interesting historical account of the dichotomy between the economics and the engineering approaches to project scheduling, see Gulledge and Womer (1986).

[2] In the context of weapons development, Peck and Scherer (1962) and Marshall and Meckling (1962) discussed these issues at length, as did Mansfield, Schnee and Wagner (1971) in a study of pharmaceuticals development. Within software engineering, a still active and very influential line of research initiated by Norden (1958) and Putnam (1978) searches for mathematical formulae which fit data on development time

this early literature reports considerable bias in forecasts of development time. This suggests that there is something inherently missing in their approach, and incentives seem to be a leading candidate.

In order to study these issues within an asymmetric information framework, this paper analyzes the question of optimal contract design in a model with time to build and adverse selection. An agent exerts effort over time towards project completion subject to a production function which is parameterized by a privately known efficiency parameter. With convex disutility of the rate of effort, the agent has an incentive to prolong the development period, ceteris paribus. The principal on the other hand, discounts the value of the project from the time of completion, and thus has an interest in speedy delivery.

The main findings of the analysis are as follows. Under complete information, the optimal accumulation of progress is such that along the efficient effort path, the marginal discounted disutility of effort is constant. That is, effort is exerted in such a manner that the agent cannot benefit from changing the intertemporal distribution of effort. In turn, optimal delivery time is such that the marginal benefit of compressing the schedule marginally is equal to the extra disutility associated with having to increase effort in order to complete marginally earlier. Turning to characterization, it is shown that an increase in the principal's gross value of the completed project decreases optimal delivery time and increases the effort exerted at all stages of the project. Furthermore, an increase in the project's scale or difficulty increases optimal delivery time and decreases exerted effort at all but the last stage of the project.

Turning to the incomplete information case, the optimal contract effectively uses delivery time as a screening device and is characterized by a *no distortion at the top* result. Namely, it is shown that the optimal contract, in trading off efficiency and informational rents, prescribes an efficient termination time only for the most efficient type of agent. For agents of lower efficiency, the optimal contract distorts delivery time by inducing inefficient delay in completion. Furthermore, rents enjoyed by the agent are decreasing in delivery time, tending to zero for the most inefficient type of agent. The comparative dynamics of the complete information model carry over to the incomplete information setting. Last, comparative analysis of the optimal contract to changes in gross project value and project scale/difficulty is performed. This analysis shows that the rents enjoyed by the agent are increasing in both the project's gross value and in its scale.

To the best of my knowledge, only two contributions have studied the issue of delivery time within a contracting framework explicitly, the first of which is Cukierman and Shiffer (1976). They study a complete information model in which a principal and an agent contract for delivery of a project. In this context, they show that many of the most commonly used payment schemes give the agent perverse incentives and are thus likely to induce inefficient delays in project completion. In their words, "in commonly used payment methods there is a built-in incentive for the supplier to submit the product at a suboptimal date". They propose a number of different ways through which contracts can induce the agent to deliver at the

and resource phasing (see e.g. Pillai and Nair, 1997 for a recent contribution to this literature and a review of existing work). Most of this work has been either descriptive or positive in nature and has few normative implications. For example, Putnam (1978) states that "if we know the management parameters [...] then we can generate the manpower, instantaneous cost, and cumulative cost of a software project at any time t by using the Rayleigh equation". In other words, the problem of scheduling is viewed in a very mechanistic way which wholly disregards any influence that contracting and incentives may have on outcomes.

optimal date. While their work identifies an important issue and goes some way in addressing it by pointing out the fallacies of existing procedures, their analysis is crucially dependent on the assumption of complete information. This assumption renders contract design almost irrelevant as the first best may be achieved by making the agent residual claimant, e.g. by simply selling the blueprint to him for a fixed fee equal to the first best profits. In practice, there seems to be many sources of asymmetric information. The agent's skills in completing the project may be private information, or the agent's accounting records may be unavailable to the principal. But whenever this type of asymmetric information is present, it is likely to be exploited for private benefit. In independent and concurrent work dealing with similar issues, Tsur and Zemel (2002) consider the timing of environmental innovations within a principal agent framework. While their model is structurally similar to the one presented here, they focus on a different set of applications and do not deal with comparative analysis of the optimal contract with respect to the parameters of the model.

Methodologically, the present work lies at the intersection of two existing literatures. First, it is in the tradition of a series of papers on optimal R&D projects, following Lucas (1971) and Kamien and Schwartz (1971). Both these contributions, as well as that of Grossman and Shapiro (1986a, 1986b), study optimal resource allocation under uncertainty as dynamic decision theoretic problems. The base model studied here is similar to the deterministic versions of these contributions. This type of model is very well suited for the analysis of completion time as it has a clear cut tradeoff between development time and resources spent (or effort exerted). Importantly, these papers do not deal with the question of incentives. Second, the incomplete information extension of the present model belongs to the literature on procurement under adverse selection, as developed by Laffont and Tirole (1986, 1998).

The basic model is set out in Section 2. In Section 3, the complete information benchmark is analyzed, and comparative dynamics analysis performed. The incomplete information case is studied in Section 4 and Section 5 offers concluding comments. The Appendix contains proofs omitted in the main text.

2. The Model

A principal owns the blueprint to a project which yields gross value $R > 0$ upon completion. This value may be interpreted as an infinite flow of future profits, accruing from the time of project completion. The reward and all expenses are discounted at rate $r > 0$. Assume that the project is completed when k units of effective effort have been accumulated. The constant k may be interpreted as a number of effective man-hours or as some physical attribute of the project, such as kilometers of a highway. Alternatively, k may be thought of as a measure of the difficulty or scale of the project.[3]

In order to complete the project, the principal hires an agent to exert effort towards project completion. Let $x(t) \in \mathbb{R}_+$ denote the agent's instantaneous rate of effort at time t. This effort is translated into progress towards project completion at rate $\theta x(t)$, where θ is distributed according to the cumulative distribution function F on $[\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} > 0$. The parameter θ characterizes the agent's efficiency.

[3]In software engineering, there are several measures used for the size of a project k. Two such measures are total *effective lines of code* and *points of functionality*. Accordingly, a widely used measure for efficiency θ is a programmer's *lines of code* per day.

In exerting effort, the agent incurs disutility at rate $\psi(x(t)) > 0$ for $x(t) > 0$ with $\psi' > 0$, $\psi'' > 0$ and $\psi''' \geq 0$. Like the principal, the agent discounts the future at rate $r > 0$. Since it will never be optimal to accumulate more effort than required to complete the project, completion time T and effort $x(t)$ satisfy the equation

$$\int_0^T \theta x(t)dt = k \tag{1}$$

Throughout, it will be assumed that the modeled problem indeed has an optimal solution, and that this solution is interior. An existence result may possibly be obtained by considering the solution of auxiliary problems as outlined in Seierstad and Sydsaeter (1987), pp. 147-151. Last, the assumption is maintained that production is worth undertaking for all types of agent efficiency θ.

3. Complete Information

Under complete information, the agent's type θ is known by the principal. Thus incentive compatibility is irrelevant and the optimal contract holds each type of agent to his reservation utility (which is normalized to zero), i.e. the participation constraint is binding. The principal must determine the optimal allocation of effort over time and the optimal time of project completion. Thus the problem of the principal is

$$\max_{x(t),T} \left\{ e^{-rT}R - \int_0^T e^{-rt}\psi(x(t))dt \right\} \tag{2}$$

subject to (1).

In analyzing this problem, it shall prove useful to make a series of transformations of the objective and the constraints. First note that (2) is equivalent to

$$\max_{x(t),T} \left\{ -\int_0^T e^{-rt}\left(rR + \psi(x(t))\right)dt \right\} \tag{3}$$

subject to (1). Second, in order to avoid working with the integral constraint (1), define the new variable $z(t) = \int_0^t \theta x(s)ds$, where $z(t)$ is just accumulated progress at time t. The new constraint for the problem is thus the following differential equation with boundary conditions:

$$\dot{z}(t) = \theta x(t), \quad z(0) = 0, \quad z(T) = k \tag{4}$$

Next, note that since the differential equation constraint in (4) holds for all t

$$\lambda(t)\left[\theta x(t) - \dot{z}(t)\right] = 0$$

for all $t \in [0, T]$ and arbitrary function $\lambda(t)$. Adding this term to the integrand in (3) does not alter the optimal solution, and the problem can therefore be rewritten as the following unconstrained problem (subject to appropriate boundary conditions in (4)):

$$\max_{x(t),T} \int_0^T \left(-e^{-rt}\left[rR + \psi(x(t))\right] + \lambda(t)\left[\theta x(t) - \dot{z}(t)\right]\right)dt =$$

$$\max_{x(t),T} \left\{ \int_0^T \left(-e^{-rt}\left[rR + \psi(x(t))\right] + \lambda(t)\theta x(t)\right)dt - \int_0^T \lambda(t)\dot{z}(t)dt \right\} \tag{5}$$

Integrating the last term in (5) by parts yields the equivalent problem

$$\max_{x(t),T}\left\{\int_0^T\left\{-e^{-rt}\left[rR+\psi(x(t))\right]+\lambda(t)\theta x(t)+\dot{\lambda}(t)z(t)\right\}dt+\lambda(0)z(0)-\lambda(T)z(T)\right\} \quad (6)$$

Treating progress $z(t)$ in (6) as a choice variable yields the familiar optimality condition for problems with an integral constraint, namely that $\dot{\lambda}(t)=0$. That is, in optimum the multiplier is constant over time and terms multiplying $\dot{\lambda}(t)$ can be omitted. Using this and using the boundary conditions (4) in (6) yields the final form of the problem:

$$\max_{x(t),T}\left\{\int_0^T\left\{-e^{-rt}\left[rR+\psi(x(t))\right]+\lambda(t)\theta x(t)\right\}dt-\lambda(T)k\right\} \quad (7)$$

Next, the problem is solved in two stages. First, the optimal effort trajectory is characterized for arbitrary termination time T. Second, the optimal termination time is characterized, given the optimal effort path.

In this problem, accumulated progress $z(t)$ is the state variable and effort $x(t)$ is the control variable. For fixed T, it follows that at an interior solution a necessary[4] condition for optimality is an effort trajectory characterized by

$$e^{rt}\theta\lambda(t)=\psi'(x(t)) \quad (8)$$

This leads to the following characterization of the optimal solution:

Proposition 1. *Under complete information: (i) Effort is exerted from the outset. (ii) Effort is strictly increasing over time.*

Proof: Part (i) follows immediately from observing that if $x(t)=0$ on some interval $[0,s]$, the whole project is just delayed for s amount of time, and the net benefit of project completion accordingly discounted. Part (ii) follows from (8) and is an artefact of discounting ■

Since the multiplier $\lambda(t)$ is constant, (8) calls for a trajectory of effort $x(t)$ such that the discounted disutility of effort is constant over the duration of the project.

Turning to the characterization of the optimal termination time, differentiating (7) with respect to termination time T and exploiting the boundary conditions, the optimal T can be characterized implicitly by the equation

$$rR+\psi(x(T))=\theta\lambda(T)e^{rT}x(T) \quad (9)$$

Evaluating (8) at $t=T$ and combining with (9) yields

$$rR+\psi(x(T))=\psi'(x(T))x(T) \quad (10)$$

[4]Note that since the second order condition with respect to effort $x(t)$ reduces to $-e^{-rt}\psi''(x(t))<0$ (the inequality follows since ψ is convex), the objective is concave in effort for fixed T and the following condition is both necessary and sufficient for optimality. It should be mentioned that (8) can be derived by Pontryagin's maximum principle, and that (9) is just the usual transversality condition. Nevertheless, the Hamiltonian function is not useful for the analysis that follows and will thus not be employed.

Equation (10) has a straightforward interpretation. Consider compressing the schedule such that termination is achieved an instant sooner. The benefit of this compression is the value of receiving the reward an instant sooner, rR, plus the costs saved at time T, namely $\psi(x(T))$. The costs of this compression is the increase in costs associated with slightly increasing effort over the (slightly shorter) duration of the project, $\psi'(x(T))x(T)$.

Before characterizing this solution further, it is shown that (9) is also a locally sufficient condition for T to be optimal. Noting that along an optimal trajectory, $dx(t)/dT = dz(t)/dT = 0$ for all $t \in [0, T]$ and using the boundary conditions in (4), the second derivative of (7) with respect to T reduces to

$$re^{-rT}\left(rR + \psi(x(T))\right) + 2\theta x(T)\lambda_T(T) \tag{11}$$

Exploiting (8) along the optimal trajectory, (11) reduces to

$$re^{-rT}\left[rR + \psi(x(T)) - \psi'(x(T))x(T)\right] - re^{-rT}\psi'(x(T))x(T) < 0$$

where the inequality follows from observing that the bracketed expression is zero at the candidate termination time T because of (10).

The comparative dynamics of the optimal solution are gathered in the following proposition, the proof of which is relegated to Appendix A.

Proposition 2. *Under complete information: (i) An increase in project value R decreases the optimal termination time T and increases the optimal path $x(t)$ at all stages of the project. (ii) An increase in project difficulty k increases the optimal termination time T, decreases initial effort $x(0)$ but leaves final effort $x(T)$ unchanged.*

The results so far may be understood by considering the marginal benefit and cost of slightly stretching the schedule in cost-termination time space, as illustrated in Figure 1.

- Insert Figure 1 here -

An increase in gross value R shifts the marginal benefit of stretching the schedule downward, while leaving unchanged the marginal cost of doing so (recall that both are negative). In turn, this shifts the intersection of the two curves (yielding the optimal delivery time) leftward. Similarly, an increase in project difficulty k leaves the marginal benefit of stretching the schedule unchanged, while pushing the marginal cost downward. The intersection thus shifts to the right. Last, note that the agent's efficiency θ only influences the marginal cost curve. An increase in θ causes the marginal cost curve to shift upward, thereby lowering the optimal termination time.

Under complete information, there are a multitude of ways in which the optimal delivery time can be implemented. The simplest is to instruct the agent to deliver at the optimal time, and to pay him a wage equal to the total discounted disutility associated with the stipulated delivery time. Alternatively, the principal may offer the agent a wage which depreciates at the same rate as the principal's value of the project. This is in effect a contract with a liquidated damages clause. This and several other equivalent contracts are suggested and discussed by Cukierman and Shiffer (1976).

Example: The derivations so far only characterize the optimal path and completion time implicitly. For the special case of quadratic disutility, i.e. $\psi(x) = x^2/2$, the optimal delivery time and effort trajectory can be derived explicitly. They are given by

$$\begin{aligned} T(\theta) &= r^{-1} \log \left[\frac{\theta\sqrt{2rR}}{\theta\sqrt{2rR} - rk} \right] \\ x(t) &= \frac{e^{rt} rk}{\theta \left(e^{rT(\theta)} - 1 \right)} \end{aligned}$$

where the condition $\underline{\theta}\sqrt{2rR} - rk > 0$ is imposed. In achieving this delivery time and keeping the agent to his participation constraint, the wage must be set to

$$w(\theta) = \frac{rk^2}{2\theta^2 \left(e^{rT(\theta)} - 1 \right)}$$

The derivation is a straightforward exercise in variational calculus and is thus omitted.[5]

4. Incomplete Information

First, note that the analysis of the complete information benchmark was carried out for arbitrary type θ and that the optimal effort trajectory and termination time is a function of the type (although this dependence was suppressed for notational clarity). Now consider the setting in which θ is not observed by the principal. In solving for the optimal contract, a choice must be made for the control variable and the state variable. As usual, the agent's rent is treated as a state variable. Since effort is a function (of time), using it as a control variable would unnecessarily complicate the analysis. But note that for arbitrary completion time T, the agent will always find it optimal to choose an effort trajectory from the efficient family given by (8). Thus, given a type θ, there is a one-to-one correspondence between the efficient trajectories $x(t)$ and termination time T. The fact that T is a sufficient statistic for effort is convenient, as termination time can thus be used as the control variable.

In solving for the optimal contract under asymmetric information, a direct revelation mechanism will be employed. In particular, the principal asks the agent to announce his type, and based on the announcement $\widehat{\theta}$ offers a contract from a menu $\{w(\widehat{\theta}), T(\widehat{\theta})\}$. The agent is then paid a wage $w(\widehat{\theta})$ and must meet the production deadline $T(\widehat{\theta})$.

It should be noted that in offering such a menu, it is implicitly assumed that the principal will not find it optimal to make wages conditional on observed progress $z(t)$. Since perfect commitment is assumed, this is indeed without loss of optimality. Were the principal to make wages contingent on progress, he would, upon observing progress, be able to perfectly infer the agent's type and thus extract all surplus. As this would be correctly anticipated by the agent, all types would pool and mimic the behavior of the least efficient agent. Clearly, this would be suboptimal from the principal's perspective, who can thus gain by committing to a contract written solely on delivery time $T(\theta)$.[6] In what follows, it is assumed that the

[5] See Toxvaerd (2002a) for details or Lucas (1971) or Kamien and Schwartz (1981) for treatments of similar problems.

[6] Furthermore, note that perfect commitment is already assumed implicitly by the use of a direct revelation mechanism.

distribution F satisfies

$$\frac{d}{d\theta}\left(\frac{F(\theta)-1}{f(\theta)}\right) \geq 0 \tag{12}$$

which is the standard monotone likelihood ratio property.[7]

4.1. The Agent's Problem. Before writing up the principal's problem under incomplete information, the relevant incentive constraints of the problem will be derived.[8] Introduce the agent's rent as a new state variable. Denote by $U(\theta,\widehat{\theta})$ the rent accruing to an agent of type θ who announces to be of type $\widehat{\theta}$. This is given by

$$U(\theta,\widehat{\theta}) = \max_{x(t)}\left\{w(\widehat{\theta}) - \int_0^{T(\widehat{\theta})} e^{-rt}\psi(x(t))dt\right\} \tag{13}$$

subject to (4) where the appropriate boundary conditions now become $z(0)=0$, $z(T(\widehat{\theta}))=k$. Following similar steps as those leading to (7), (13) can be rewritten as

$$U(\theta,\widehat{\theta}) = \max_{x(t)}\left\{w(\widehat{\theta}) + \int_0^{T(\widehat{\theta})} \left\{-e^{-rt}\psi(x(t)) + \lambda(t)\theta x(t)\right\} dt - \lambda(T(\widehat{\theta}))k\right\} \tag{14}$$

In Appendix B it is shown that incentive compatibility implies that $T(\theta)$ is a monotone (decreasing) function, which in turn implies that the function $w(\widehat{\theta})$ is differentiable. The use of the first-order approach is thus justified, and the first order condition for truth-telling is given by $U_{\widehat{\theta}}(\theta,\widehat{\theta})=0$, while the second order condition is $U_{\widehat{\theta}\widehat{\theta}}(\theta,\widehat{\theta})<0$. Totally differentiating the first order condition yields $U_{\widehat{\theta}\widehat{\theta}}(\theta,\widehat{\theta}) = -U_{\widehat{\theta}\theta}(\theta,\widehat{\theta})$, and thus the local second order condition becomes $U_{\widehat{\theta}\theta}(\theta,\widehat{\theta})>0$. From (14) it follows that[9]

$$U_{\widehat{\theta}\theta}(\theta,\theta) = -e^{-rT(\theta)}rk\theta^{-2}\psi'(x(T(\theta)))\dot{T}(\theta) \tag{15}$$

and so the local second order condition reduces to $\dot{T}(\theta)<0$. In the optimal contract it is indeed the case that $\dot{T}(\theta)<0$. This is shown in Appendix D, where it is also shown that this monotonicity result and the first order condition for truth-telling are sufficient conditions for truth-telling to be a global optimum.

Denote by $U(\theta)$ the rent accruing to an agent of type θ when truth-telling is induced, i.e. $U(\theta) = U(\theta,\widehat{\theta})|_{\widehat{\theta}=\theta}$. By the envelope theorem, it follows from (14) that

$$\dot{U}(\theta) = \int_0^{T(\theta)} e^{-rt}\theta^{-1}\psi'(x(t))x(t)dt \geq 0 \tag{16}$$

where (8) has been used to eliminate the multiplier. This differential equation in θ shows the rate of change in the agent's rent when truth-telling is induced.

[7] See e.g. Laffont and Tirole (1998) for an interpretation.

[8] As will become apparent in what follows, the analysis of the optimal incomplete information contract will employ arguments that parallel those by Laffont and Tirole (1986). But the analysis is greatly complicated by the fact that the agent's type θ will appear in limits of integration and in the constraints. In Laffont and Tirole (1986), the target (i.e. costs) is linear in both type and effort, a fact that greatly simplifies their analysis. Thus, it is not a priori clear that their results can be carried over to the current setting.

[9] See Appendix C for details of this derivation.

4.2. The Principal's Problem. Next turn to the principal's maximization problem. It is given by

$$\max_{x(t),T(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \int_0^{T(\theta)} \left(-e^{-rt}[rR + \psi(x(t))] + \zeta(t)\theta x(t)\right) dt - \zeta(T(\theta))k - U(\theta) \right\} f(\theta)d\theta \quad (17)$$

subject to the participation constraint $U(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and the incentive compatibility constraint (16). In order to avoid confusion with the multiplier $\lambda(t)$ in the complete information analysis, $\zeta(t)$ in (17) denotes the multiplier on the differential equation constraint in (4). Since from (16) the agent's rent is increasing in type, the participation constraint can be reduced to the requirement that $U(\underline{\theta}) \geq 0$. The problem can be further simplified by noting that (16) implies that

$$\mu(\theta) \left[\int_0^{T(\theta)} e^{-rt}\theta^{-1}\psi'(x(t))x(t)dt - \dot{U}(\theta) \right] = 0 \quad (18)$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and arbitrary function $\mu(\theta)$. Adding (18) to the integrand (in θ) in (17) does not alter the optimal solution. Integrating the last term in (18) by parts and using the boundary conditions in (4), the principal's problem can be reduced to the following unconstrained problem:

$$\max_{x(t),T(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \{\pi(\theta) - [\zeta(T(\theta))k + U(\theta)]f(\theta) + \dot{\mu}(\theta)U(\theta)\} \, d\theta - \mu(\overline{\theta})U(\overline{\theta}) \quad (19)$$

where

$$\pi(\theta) = \int_0^{T(\theta)} \left(\left[-e^{-rt}\left(rR + \psi(x(t))\right) + \zeta(t)\theta x(t)\right] f(\theta) + \mu(\theta)e^{-rt}\theta^{-1}\psi'(x(t))x(t) \right) dt$$

Note that the problem now has an added state variable, namely the agent's rent $U(\theta)$. Treating it as a choice variable yields $\dot{\mu}(\theta) = f(\theta)$. But since in optimum $U(\underline{\theta}) = 0$, this means that $\mu(\theta) = F(\theta) - 1 \leq 0$.

4.3. Characterization. From (19) it follows that the optimal effort trajectory is characterized by

$$e^{rt}\theta\zeta(t) = \psi'(x(t)) - \frac{F(\theta) - 1}{f(\theta)} \left[\psi''(x(t))x(t) + \psi'(x(t))\right] \theta^{-1} \quad (20)$$

Comparing (20) with (8), it is immediate that the optimal trajectory under incomplete information is lower than that under complete information. As will be shown shortly, this naturally also influences the optimal delivery time $T(\theta)$.

To characterize the optimal termination time, differentiate (19) with respect to $T(\theta)$ and evaluate along the optimal trajectory to obtain

$$e^{rT(\theta)}\zeta(T(\theta))\theta x(T(\theta)) = rR + \psi(x(T(\theta))) - \frac{F(\theta) - 1}{f(\theta)}\psi'(x(T(\theta)))x(T(\theta))\theta^{-1} \quad (21)$$

Evaluating (20) at $t = T(\theta)$ and combining with (21) yields

$$rR + \psi(x(T(\theta))) = \psi'(x(T(\theta)))x(T(\theta)) - \frac{F(\theta) - 1}{f(\theta)} \left[\psi''(x(T(\theta)))x(T(\theta))^2\right] \theta^{-1} \qquad (22)$$

Equation (22) characterizes the optimal termination time $T(\theta)$ implicitly. It calls for setting $T(\theta)$ such that the marginal benefit of completing an instant sooner is equated with the marginal cost of doing so. But note that the right hand side of (22) has a positive term added as compared to the right hand side of (10). This term captures the fact that in trading off efficiency and informational rents, the optimal incomplete information contract calls for a distortion of delivery time. In cost-termination time space, as illustrated in Figure 1, asymmetric information leaves the marginal benefit of stretching the schedule unchanged, while shifting downward the marginal costs of doing so. In turn, the intersection of the curves moves rightward. Also, note that the shift caused by asymmetric information is increasing in type θ. To see the direction of this distortion, note that for $\theta = \overline{\theta}$, $F(\overline{\theta}) = 1$ and thus $rR + \psi(x(T(\overline{\theta}))) = \psi'(x(T(\overline{\theta})))x(T(\overline{\theta}))$. Thus there is no distortion of delivery time for the most efficient type of agent. In contrast, for $\theta < \overline{\theta}$, the marginal cost of an instant's postponement is greater than the marginal benefit of such a postponement, and thus the prescribed delivery time is greater than the efficient one.[10]

Next, from (16), an expression for the agent's rent as a function of type θ may be obtained. Integrating yields

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \int_0^{T(\alpha)} e^{-rt} \alpha^{-1} \psi'(x(t)) dt d\alpha + U(\underline{\theta}) \qquad (23)$$

For $\theta = \underline{\theta}$ there is no rent, i.e. $U(\underline{\theta}) = 0$, while for $\theta > \underline{\theta}$, rents are positive and increasing.

The results so far are gathered in the following proposition:

Proposition 3. *Under incomplete information: (i) The optimal contract induces inefficient delay in delivery for all but the most efficient type of agent, and the distortion is decreasing in type. (ii) The least efficient agent earns no rents while more efficient types earn positive rents which are increasing in type.*

Example (continued): In the example considered in Section 3, let $\theta \in \{\underline{\theta}, \overline{\theta}\}$ and $q = \Pr(\theta = \overline{\theta})$. Optimal delivery times are given by

$$\begin{aligned} T(\overline{\theta}) &= r^{-1} \log \left[\frac{\overline{\theta}\sqrt{2rR}}{\overline{\theta}\sqrt{2rR} - rk} \right] \\ T(\underline{\theta}) &= r^{-1} \log(\rho) \end{aligned}$$

where ρ is the relevant solution to the polynomial

$$\alpha(\rho - 1)^2 + \beta\rho^2(\rho - 1)^2 + \gamma\rho^2 = 0$$

[10] Note that in contrast to Laffont and Tirole (1986), it is not appropriate to talk about under provision of effort within this model, as the total amount of effective effort needed for project completion is exogenously given. Rather, the optimal contract induces an inefficient distribution of effort over time.

and

$$\alpha = 2r(1-q)R, \quad \beta = q\left(\frac{\overline{\theta}\sqrt{2rR}-rk}{\overline{\theta}}\right)^2, \quad \gamma = -\left(\frac{rk}{\underline{\theta}}\right)^2$$

Clearly, for the high type $\overline{\theta}$ delivery time coincides with the efficient one. For $\underline{\theta}$, it can be shown numerically that $T(\underline{\theta})$ is larger than the efficient delivery time for the low type.[11]

4.4. Comparative Analysis. Next, turn to the comparative analysis of the optimal contract. In analyzing the complete information benchmark, comparative dynamics analysis was presented for variations in project difficulty k and gross project value R. These carry over to the incomplete information setting.

Proposition 4. *Under incomplete information: (i) An increase in project value R decreases the optimal termination time T and increases the optimal path $x(t)$ at all stages of the project. (ii) An increase in project difficulty k increases the optimal termination time T, decreases initial effort $x(0)$ but leaves final effort $x(T)$ unchanged.*

The proof of this proposition is similar to that of Proposition 2 and is thus omitted.[12]

Last, note that the rent of the agent as given by (23) is naturally also affected by such change in the parameters of the project. Specifically, the rent changes in the following way:

Proposition 5. *Under incomplete information: (i) An increase in project value R increases the rent earned by all but the least efficient type of agent. (ii) An increase in project difficulty k increases the rent for all but the least efficient type of agent.*

Proof: (i) Recall that optimal termination time is decreasing in R. But $\lambda_T < 0$ (see Appendix A) and thus λ increases as an effect of increasing R. By (14) the agent's rent therefore decreases, ceteris paribus. In order not to violate the participation constraint of type $\underline{\theta}$, the wage specified for the most inefficient type of agent must increase. Denote this increase by $\Delta w(\underline{\theta})$. Suppose that $\Delta w(\theta) \leq \Delta w(\underline{\theta})$ for some type θ slightly above $\underline{\theta}$. If the wage increase is largest for the least efficient agent, the agent with slightly higher type can benefit from reporting to be type $\underline{\theta}$, thereby violating the incentive compatibility constraint. Thus $\Delta w(\theta)$ is increasing in θ. But since $\lambda_\theta < 0$ (see Appendix A), the cost of completing marginally earlier is decreasing in type. Since the change in wages is increasing in type and the cost is decreasing, the result follows. (ii) From (14), it follows that $U(\theta)_k = -\lambda(T(\theta)) < 0$. Again, note that in increasing difficulty, the wage specified for the least efficient agent must increase to the extent that it yields zero rent after the change. Suppose that $\Delta w(\theta) \leq \Delta w(\underline{\theta})$ for some θ slightly above $\underline{\theta}$, i.e. that the increase in wage is bounded above by that prescribed to the least efficient agent. Ex ante, i.e. before the increase in k, incentive compatibility implies that any type prefers reporting the truth to mimicking that of a lower type. Last note that since $\lambda_\theta(T(\theta)) < 0$ (see Appendix A), the disutility associated with higher k is

[11] The derivation of the optimal completion times is very lengthy and does not add much new insight. They are thus omitted. For details, see Toxvaerd (2002a).

[12] For the proof, define the function $\xi(x(t)) \equiv \psi'(x(t)) - \frac{F(\theta)-1}{f(\theta)}\left[\psi''(x(t))x(t) + \psi'(x(t))\right]\theta^{-1}$ and note that $\xi'(x(t)) > 0$ since $\psi''' \geq 0$. Thus $\xi(x(t))$ is monotone and therefore invertible. Substituting $\xi(x(t))$ for $\psi'(x(t))$, all the proofs of Appendix A follow immediately.

decreasing in type. If $\Delta w(\theta) \leq \Delta w(\underline{\theta})$ as supposed, incentive compatibility is violated after the increase in k as the type slightly higher than $\underline{\theta}$ can now secure himself a higher increase in wage by reporting to be type $\underline{\theta}$. Thus it must be that $\Delta w(\theta) > \Delta w(\underline{\theta})$. Since the increase in wage is thus increasing in type but the disutility decreasing in type, rent must by (14) increase ■

For completeness, note that in the optimal contract, it follows from (23) that the wage transfer as a function of the agent's type is given by

$$w(\theta) = \int_{\underline{\theta}}^{\theta} \int_{0}^{T(\alpha)} e^{-rt} \alpha^{-1} \psi'(x(t)) dt d\alpha + \int_{0}^{T(\theta)} e^{-rt} \psi(x(t)) dt$$

5. Concluding Remarks

The present paper offers a stylized model in which optimal delivery times and deadlines can be derived and fruitfully analyzed. While it is certainly true that in practice a myriad of other unmodeled factors influence how fast a project is completed or at which rate progress towards completion is achieved, the basic tradeoff between effort and completion time emphasized in the present work seems to be of main importance. Not only is it satisfying on theoretical grounds, but is has also been identified in the empirical literature.[13] While the introductory section motivated the model in terms of a procurement relationship, the analysis is also applicable to relationships involving workers, managers or subdivisions within a firm.

In the present analysis, it is assumed that the principal is uncertain about the agent's type θ. An alternative, but qualitatively similar, set of assumptions would be that θ is known by the principal but that the agent has private information about the scale of the project k. This variation more closely resembles the setup in Grossman and Shapiro (1986b).

An interesting, but significant, extension of the current analysis is to assume that θ, instead of characterizing the agent, is a random variable evolving according to a Brownian motion. Under the assumption that progress $z(t)$ is observable but that both the sample path of $\theta(t)$ and the effort path $x(t)$ are unobservable, the model is effectively transformed to one of continuous time moral hazard. This problem has a close parallel in the model studied by Holmstrom and Milgrom (1987). In that model, the duration of the relationship (i.e. the time interval) is fixed while the cumulated effort (or the area under the effort curve) is endogenous. In the model suggested here, the amount of cumulated effort is fixed, while the time interval is endogenous. While insights from their analysis may be of use in analyzing such an extension, the correspondence is not straightforward as their analysis is confined to a very specific utility function and discounting is not considered. Clearly, discounting lies at the heart of the suggested extension.

A simpler way of introducing moral hazard into a model of deadlines is presented in Toxvaerd (2002b). In that (discrete time) model, an agent moves along the nodes of a directed graph, with transition probabilities determined by the agent's unobserved effort. That model is also better suited to study the effects that commitment have on project completion, and to study dynamics of optimal contracting.

[13] See e.g. the discussion in Peck and Sherer (1962), where this tradeoff is derived from an analysis of manhour-curves, and the literature surveyed in Pillai and Nair (1997) which studies so-called Norden-Raleigh analysis of expenditure patterns and project scheduling.

6. Appendix A

In order to derive the comparative dynamics of the model, some intermediate results are needed.[14] First note that in optimum, the multiplier $\lambda(t)$ is determined by (8), (9) and the constraint (1). Thus it solves the equation

$$\int_0^T \theta(\psi')^{-1}\left[e^{rt}\theta\lambda(t)\right]dt - k = 0$$

Keep in mind that the solution is a function of k and θ for arbitrary termination time T. By the implicit function theorem, this value is unique and has continuous derivatives. Specifically,

$$\begin{aligned}
\lambda(t)_k &= \frac{1}{\theta^2\int_0^T((\psi')^{-1})'(e^{rt}\theta\lambda(t))dt} > 0 \\
\lambda(t)_T &= \frac{-(\psi')^{-1}(e^{rt}\theta\lambda(t))}{\theta\int_0^T((\psi')^{-1})'(e^{rt}\theta\lambda(t))dt} < 0 \\
\lambda(t)_\theta &= \frac{-\int_0^T[(\psi')^{-1}(e^{rt}\theta\lambda(t)) + \theta((\psi')^{-1})'(e^{rt}\theta\lambda(t))e^{rt}\lambda(t)]dt}{\theta^2\int_0^T((\psi')^{-1})'(e^{rt}\theta\lambda(t))dt} < 0
\end{aligned}$$

Next, define the value function

$$V(x(t),T;R,k,\theta) = \max_{x(t),T}\left\{\int_0^T\left\{-e^{-rt}\left[rR+\psi(x(t))\right]+\lambda(t)\theta x(t)\right\}dt - \lambda(T)k\right\} \tag{24}$$

Differentiating (9) with respect to R, employing the envelope theorem and rearranging yields

$$\frac{dT}{dR} = \frac{-V_{TR}(x(t),T;R,k,\theta)}{V_{TT}(x(t),T;R,k,\theta)} = \frac{re^{-rT}}{V_{TT}(x(t),T;R,k,\theta)} < 0$$

Initial effort is given by $\psi'(x(0)) - \lambda = 0$. Thus

$$\frac{dx(0)}{dR} = \psi''(x(0))^{-1}\lambda_T\frac{dT}{dR} > 0$$

Terminal effort is in turn determined implicitly by (10), i.e. $rR = \psi'(x(T))x(T) - \psi(x(T))$. An increase in R must correspond to an increase in the right hand side. But this is increasing in terminal effort since

$$\frac{d}{dx(T)}[\psi'(x(T))x(T) - \psi(x(T))] = \psi''(x(T))x(T) > 0$$

Thus

$$\frac{dx(T)}{dR} > 0$$

In sum, higher gross value R reduces optimal completion time and increases effort at all stages of the project.

[14] These derivations are along the lines of those in Grossman and Shapiro (1986a).

Next, consider an increase in the difficulty of the project k. Differentiating (24) with respect to k and then with respect to T and rearranging yields

$$\begin{aligned}\frac{dT}{dk} &= \frac{-V_{Tk}(x(t),T;R,k,\theta)}{V_{TT}(x(t),T;R,k,\theta)} \\ &= \frac{-V_{kT}(x(t),T;R,k,\theta)}{V_{TT}(x(t),T;R,k,\theta)} \\ &= \frac{\lambda_T(t)\theta^{-1}}{V_{TT}(x(t),T;R,k,\theta)} > 0\end{aligned}$$

Next, since k does not appear in (10),

$$\frac{dx(T)}{dk} = 0$$

Thus, higher difficulty increases optimal completion time, and leaves terminal effort unchanged. To see how initial effort varies with k, note that the following identity holds:

$$V(x(t),T;R,k_1+k_2,\theta) = V(x(t),T;V(x(t),T;R,k_2,\theta),k_1,\theta) \tag{25}$$

That is, a project with return R of difficulty $k_1 + k_2$ is worth as much as a project with return $V(x(t),T;R,k_2,\theta)$ of difficulty k_1. Differentiating (25) with respect to k_1 yields

$$V_k(x(t),T;R,k_1+k_2,\theta) = V_k(x(t),T;V(x(t),T;R,k_2,\theta),k_1,\theta)$$

Differentiating next with respect to k_2 yields

$$V_{kk}(x(t),T;R,k_1+k_2,\theta) = V_{kR}(x(t),T;V(x(t),T;R,k_2,\theta),k_1,\theta)V_k(x(t),T;R,k_2,\theta)$$

But $V_k(x(t),T;R,k_2,\theta) = -\lambda < 0$, and thus

$$sign\,[V_{kk}(x(t),T;R,k_1+k_2,\theta)] = -sign\,[V_{kR}(x(t),T;V(x(t),T;R,k_2,\theta),k_1,\theta)]$$

Last, since

$$V_{kR}(x(t),T;R,k_2,\theta) = -\lambda_R = -\lambda_T\frac{dT}{dR} < 0$$

it follows that

$$V_{kk}(x(t),T;R,k_1+k_2,\theta) > 0$$

Thus the value function is a decreasing convex function of difficulty k. That implies that

$$V_{kk}(x(t),T;R,k_2,\theta) = -\lambda_k > 0 \Rightarrow \lambda_k < 0$$

Since initial effort is given implicitly by the equation $\psi'(x(0)) - \lambda = 0$, it follows that

$$\frac{dx(0)}{dk} = \psi''(x(0))^{-1}\lambda_k < 0$$

In other words, an increase in the difficulty of the project lowers initial effort in the optimal path.

As an aside, note that by the envelope theorem,

$$V_R(x(t), T; R, k, \theta) = e^{-rT}$$

and thus

$$V_{RR}(x(t), T; R, k, \theta) = -re^{-rT}\frac{dT}{dR} > 0$$

Thus the value function is increasing and convex in the gross value of the project R.

7. Appendix B

To show monotonicity of $T(\theta)$ in θ (i.e. that it is decreasing), pick two types $\underline{\theta}, \overline{\theta}$ with $\underline{\theta} < \overline{\theta}$ and notice that incentive compatibility implies that

$$\begin{aligned} U(\overline{\theta}, \overline{\theta}) &\geq U(\overline{\theta}, \underline{\theta}) \\ U(\underline{\theta}, \underline{\theta}) &\geq U(\underline{\theta}, \overline{\theta}) \end{aligned} \tag{26}$$

Substracting and rearranging yields

$$\left[U(\overline{\theta}, \overline{\theta}) - U(\underline{\theta}, \overline{\theta})\right] - \left[U(\overline{\theta}, \underline{\theta}) - U(\underline{\theta}, \underline{\theta})\right] \geq 0$$

Using (14), this implies that

$$\begin{aligned} & \left[\int_0^{T(\overline{\theta})} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\overline{\theta}x(t)\right) dt - \lambda(T(\overline{\theta}))k\right] \\ & - \left[\int_0^{T(\overline{\theta})} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\underline{\theta}x(t)\right) dt - \lambda(T(\overline{\theta}))k\right] \\ & - \left[\int_0^{T(\underline{\theta})} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\overline{\theta}x(t)\right) dt - \lambda(T(\underline{\theta}))k\right] \\ & + \left[\int_0^{T(\underline{\theta})} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\underline{\theta}x(t)\right) dt - \lambda(T(\underline{\theta}))k\right] \\ \geq \; & 0 \end{aligned} \tag{27}$$

Differentiating (14) with respect to $T(\theta)$, using (8) and differentiating again with respect to θ yields

$$U_{T(\theta)\theta}(\theta, \theta) = -e^{-rT(\theta)}rk\theta^{-2}\psi'(x(T(\theta)))$$

which in turn implies that (27) is equivalent to

$$-\int_{T(\underline{\theta})}^{T(\overline{\theta})} \int_{\underline{\theta}}^{\overline{\theta}} e^{-r\beta}rk\alpha^{-2}\psi'(x(\beta))d\alpha d\beta \geq 0$$

Since $\overline{\theta} > \underline{\theta}$, necessarily $T(\overline{\theta}) < T(\underline{\theta})$. Next, note that (26) and (14) imply that for $\widehat{\theta} > \theta$,

$$\begin{aligned} & \left[\int_0^{T(\widehat{\theta})} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\theta x(t)\right) dt - \lambda(T(\widehat{\theta}))k\right] \left(\widehat{\theta} - \theta\right)^{-1} \\ & - \left[\int_0^{T(\theta)} \left(-e^{-rt}\psi(x(t)) + \lambda(t)\theta x(t)\right) dt - \lambda(T(\theta))k\right] \left(\widehat{\theta} - \theta\right)^{-1} \\ \geq \; & \frac{w(\widehat{\theta}) - w(\theta)}{\widehat{\theta} - \theta} \end{aligned}$$

For $\widehat{\theta} \to \theta$, the left hand side of this inequality converges to

$$\dot{T}(\theta)\left[e^{-rT(\theta)}\psi(x(T(\theta))) - \theta\lambda(T(\theta))x(T(\theta)) + \lambda_T(T(\theta))k - \lambda_T(T(\theta))\int_0^{T(\theta)} \theta x(t)dt\right]$$

which, after exploiting (8) and evaluating along the optimal trajectory reduces to

$$\dot{T}(\theta)\left[\psi(x(T(\theta))) - \psi'(x(T(\theta)))x(T(\theta))\right] e^{-rT(\theta)}$$

A similar argument holds for $\widehat{\theta} \to \theta$ with $\widehat{\theta} < \theta$. Since these limits coincide, $w(\theta)$ is differentiable at a point where the two last terms of the integrand in (14) are differentiable.

8. Appendix C

Differentiating (14) with respect to announced type $\widehat{\theta}$ and evaluating along the optimal trajectory $x(t)$ yields

$$U_{\widehat{\theta}}(\theta, \widehat{\theta}) = w(\widehat{\theta}) - \dot{T}(\widehat{\theta}) \begin{pmatrix} e^{-rT(\widehat{\theta})}\psi(x(T(\widehat{\theta}))) - \theta x(T(\widehat{\theta}))\lambda(T(\widehat{\theta})) \\ +\lambda_T(T(\widehat{\theta}))k - \lambda_T(T(\widehat{\theta}))\int_0^{T(\widehat{\theta})} \theta x(t)dt \end{pmatrix} = 0 \qquad (28)$$

Next, exploit (8) to eliminate terms in (28) involving the multiplier. Differentiating the resulting expression with respect to type θ and evaluating at $\widehat{\theta} = \theta$ yields (15).

9. Appendix D

To show that $\dot{T}(\theta) < 0$, start by differentiating (19) with respect to $T(\theta)$. In the resulting expression, exploit (20) to eliminate terms involving the multiplier $\zeta(t)$. Once this substitution is completed, differentiate with respect to type θ and simplify by evaluating along the optimal trajectory $x(t)$. Rearranging yields

$$\dot{T}(\theta) = \frac{N}{D}$$

where

$$\begin{aligned} N &= -\theta\psi'(x(T(\theta)))x(T(\theta)) + \left(\frac{F(\theta)-1}{f(\theta)}\right)\left[\psi'(x(T(\theta)))x(T(\theta)) + \psi''(x(T(\theta)))x(T(\theta))^2\right] \\ D &= kr\theta\psi'(x(T(\theta))) + \frac{d}{d\theta}\left(\frac{F(\theta)-1}{f(\theta)}\right)\theta^2\psi''(x(T(\theta)))x(T(\theta))^2 \\ & \quad - \left(\frac{F(\theta)-1}{f(\theta)}\right)\left[kr\psi'(x(T(\theta))) + kr\psi''(x(T(\theta)))x(T(\theta)) + \theta\psi''(x(T(\theta)))x(T(\theta))^2\right] \end{aligned}$$

While the numerator N is unambiguously negative, the denominator D is positive under assumption (12).

10. Appendix E

Given that

$$\begin{aligned} U_{\widehat{\theta}}(\theta,\theta) &= 0 \\ \dot{T}(\theta) &< 0 \end{aligned}$$

hold, truth-telling, i.e. $\widehat{\theta} = \theta$ is a global optimum. To see this, assume that type θ strictly prefers to report type $\widehat{\theta} \neq \theta$ so

$$U(\theta, \widehat{\theta}) > U(\theta, \theta)$$

This implies that $U(\theta, \widehat{\theta}) - U(\theta, \theta) > 0$ and thus

$$\int_{\theta}^{\widehat{\theta}} U_{\widehat{\theta}}(\theta, \alpha) d\alpha > 0$$

Using the first order condition for truth-telling, this inequality is equivalent to

$$\int_{\theta}^{\widehat{\theta}} \left[U_{\widehat{\theta}}(\theta, \alpha) - U_{\widehat{\theta}}(\alpha, \alpha) \right] d\alpha > 0$$

or

$$\int_{\theta}^{\widehat{\theta}} \int_{\alpha}^{\theta} U_{\widehat{\theta}\theta}(\beta, \alpha) d\beta d\alpha > 0$$

Using (15), this yields

$$-\int_{\theta}^{\widehat{\theta}} \int_{\alpha}^{\theta} e^{-rT(\alpha)} rk\beta^{-2} \psi'(x(T(\alpha))) \dot{T}(\alpha) d\beta d\alpha > 0$$

Suppose $\widehat{\theta} > \theta$. Since $\dot{T}(\theta) < 0$, then necessarily $\theta \geq \alpha$ for all $\alpha \in [\theta, \widehat{\theta}]$. Specifically, pick $\alpha = \widehat{\theta}$ which implies that $\widehat{\theta} > \theta \geq \alpha = \widehat{\theta}$, a contradiction. Similarly, suppose $\widehat{\theta} < \theta$ which implies that $\theta \leq \alpha$ for all $\alpha \in [\widehat{\theta}, \theta]$. Pick $\alpha = \widehat{\theta}$, which implies that $\widehat{\theta} < \theta \leq \alpha = \widehat{\theta}$, a contradiction. Thus there is no announcement $\widehat{\theta} \neq \theta$ which yields a strictly higher utility for the agent.

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Figure 1. Benefits and costs as functions of termination time