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BOUNDED RATIONALITY AND SOCIALLY OPTIMAL LIMITS ON CHOICE IN A SELF-SELECTION MODEL

by

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Bounded Rationality and Socially Optimal Limits on Choice in A Self-Selection Model

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Abstract

When individuals choose from whatever alternatives available to them the one that maximizes their utility then it is always desirable that the government provide them with as many alternatives as possible. Individuals, however, do not always choose what is best for them and their mistakes may be exacerbated by the availability of options. We analyze self-selection models, when individuals know more about themselves than it is possible for governments to know, and show that it may be socially optimal to limit and sometimes to eliminate individual choice. As an example, we apply Luce's (1959) model of random choice to a work-retirement decision model and show that the optimal provision of choice is positively related to the degree of heterogeneity in the population and that even with very small degrees of non-rationality it may be optimal not to provide individuals any choice.

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1 Introduction

Providing choice to individuals is beneficial because it can satisfy people's varied tastes. Often, however, expanded choices lead to decision errors. Psychological studies of decision-making suggest that these errors and other costs associated with the choice process[1] can be significant and sometime outweigh the benefits conferred by a larger set of alternatives. This happens particularly in areas where decision makers lack expertise and their evaluation requires complex calculations. Old-age pension programs is a relevant example. Savings for consumption during retirement involves many interrelated uncertainties. Long-ranged discounting and annuitization depend critically on personally uncertain attributes such as health, mortality rates and future earnings, as well as macro variables such as interest rates. It is difficult to evaluate these uncertainties early in life as they unfold long after decisions are made.

Insurance policies offered by government cannot be based on individual characteristics because individuals do not willingly reveal their full characteristics. The government can reasonably know the distribution of characteristics within the population and will devise its policy to best fit this distribution, taking into account individuals' self-selection among alternative programs available to all. This '*Second-Best*' social optimum typically provides for a range of choice to individuals. It is interesting to inquire whether relaxing the assumption of fully rational choice by individuals due to reasons such as those discussed above, removes or limits the presumption that provision of choice is socially desirable. This question was first raised by Mirrlees (1987), who provided some insightful examples of non-rationality leading to socially optimal policy which leaves no choice to individuals.

[1] Lowenstein (2000) identifies three types of costs associated with the evaluation of alternatives: the opportunity costs of the *time* it takes; the tendency to make *errors* under decision 'overload' and the *psychic* costs of anxiety and regret. He cites a number of studies showing that when decisions become complex, people tend to procrastinate and to revert to simple but sub-optimal decision rules, such as choosing the 'default' alternative.

Luce (1959) proposed a model of '*bounded-rationality*' in which individuals attempt to maximize utility but make errors in the decision process. These errors lead to random deviations of the chosen alternative from the best, reflecting utility maximization imperfectly. This is the well-known *Logit model*. One merit of the model is that it allows a convenient measure of the degree of non-rationality, which is the focus of our analysis.

In this paper we apply Luce's model to a situation in which individuals have to choose between work and no-work (retirement). When offered consumption levels for workers and non-workers, individuals with varying degree of labor disutility probabilistically self-select themselves between work and non-work. The government's objective is to choose consumption levels that maximize a utilitarian social welfare function subject to a resource constraint. The optimal policy depends on individuals' degree of non-rationality. When fully rational, it is always optimal to offer them choice. However, we demonstrate that with some degree of non-rationality, the elimination of the retirement (or work) option may be socially optimal. Calculations based on standard functional forms demonstrate that this elimination of choice becomes optimal at surprisingly low levels of non-rationality.

The government is assumed to pursue a socially optimal policy. With less than rational behavior by the government, it may be desirable to design a constitution which limits the choices available to such government. This, in turn, may further restrict the provision of choice to individuals. We plan to study this question in a subsequent paper.

In the context of the current debate about reforming social security with provision of more choice to individuals, our calculations suggest that benefit-cost assessments of alternative reforms should incorporate a careful analysis of individuals' potential errors and misperceptions.

2 The Luce Model[2]

Consider an individual who has to choose one among a set of mutually exclusive alternatives. Neoclassical economic theory assumes that the individual has a utility function that allows him or her to rank these alternatives, choosing the highest ranked. Psychologists (e.g. Luce (1959), Tversky (1969) and (1972)) criticized this deterministic approach, arguing that the outcome should be viewed as a probabilistic process. Their approach is to view utility as deterministic but the choice process to be probabilistic. The individual does not necessarily choose the alternative that yields the highest utility and instead has a probability of choosing each of the various possible alternatives.

A model of '*bounded rationality*' along these lines has been proposed by Luce (1959). Luce shows that when choice probabilities satisfy a certain axiom (the '*choice axiom*'), a scale, termed 'utility', can be defined over alternatives such that the choice probabilities can be derived from the scales ('utilities') of the alternatives.

Consider a set S of a finite number, n, of discrete alternatives, a_i, $i = 1, 2..., n$. Luce's (*Multinomial) Logit Model* postulates that the probability that an individual chooses some alternative $a_i \in S$, $p_S(a_i)$, is given by

$$p_i = p_S(a_i) = \frac{e^{qu_i}}{\sum\limits_{j=1}^{n} e^{qu_j}}, \quad i = 1, 2..., n \tag{1}$$

where $u_i = u(a_i)$ for some real-valued utility function u, and q is a positive constant representing the *'precision'* of choice. When $q = 0$, all alternatives

[2] For a comprehensive discussion of discrete choice theory see Anderson, de-Palma and Thisse (1992).

have an equal probability to be chosen: $p_i = \frac{1}{n}$ for all $i = 1, 2..., n$. As q increases to $+\infty$, p_i increases monotonically, approaching 1, when u_i is the largest among all u_j, $j = 1, 2, ..., n$, and decreases monotonically, approaching 0, otherwise. Thus, it is natural to call the parameter q the *'degree of rationality'* (with $q = \infty$ called *'perfect rationality'*).[3]

[3]Debreu (1990) has pointed out a weakness in Luce's model. The introduction of a new alternative "more than proportionately reduces the choice probabilities of existing alternatives that are similar, while causing less than proportionate reductions in the choice probabilities of dissimilar alternatives" (Anderson, *et-al* (1992)). This is the well-known "blue bus/red bus" paradox. This objection, can be pertinent in imperfectly competitive market circumstances, when firms may take advantage of this implication. In the social welfare context, on the other hand, the government may take advantage of this property in providing weight to socially desirable 'default' alternatives. Generally, Luce's model has considerable merit as it incorporates a tendency to utility maximization and the parameter q provides a convenient measure of the degree of rationality.

3 Social Welfare Analysis

Suppose that the population consists of heterogeneous individuals, each characterized by a parameter θ. Individual θ's utility of alternative i is $u_i(\theta) = u(a_i, \theta)$, with the corresponding choice probabilities $p_i(\theta)$ given by (1), $i = 1, 2, ..., n$. We postulate (Mirrlees (1987)) that individuals' welfare is represented by expected utility, $V(\theta)$,

$$V(\theta) = \sum_{i=1}^{n} p_i(\theta) u_i(\theta) \tag{2}$$

We assume that all individuals have the same q[4] and that social welfare, W, is utilitarian:

$$W(q) = \int V(q) dF(\theta) = \int \left[\sum_{i=1}^{n} e^{qu_i} u_i \Big/ \sum_{j=1}^{n} e^{qu_j} \right] dF(\theta) \tag{3}$$

where $F(\theta)$ is the distribution function of θ in the population. The government's objective is to choose policies that maximize W subject to relevant resource constraints and taking into account individual reactions to alternative policies. We begin by assuming that individuals are passive, i.e. they only choose alternatives probabilistically as described above and there are no resource constraints[5].

[4] It is interesting to analyze the case of different group levels of q. Individual 'awareness' of their own and of others' levels of q and the corresponding market interactions and strategies become then crucial issues.

[5] It is easy to incorporate resource constraints when they are at the individual level. Thus, suppose that when choosing alternative i, individual θ's utility is $u_i(x_i, y, \theta)$, where x_i is an attribute of alternative i to be chosen by the individual and y is the quantity of a numeraire good. With given income $R(\theta)$ and a private cost $c_i(x_i, \theta)$ of alternative i, $y = R(\theta) - c_i(x_i, \theta)$ (≥ 0). The utility $u_i(\theta)$ in the text can be interpreted as the *optimized* utility ($w.r.t.\, x_i$) which incorporates the resource constraint. See below on the case when there is an aggregate resource constraint (and hence subject to government policy).

Let $\overline{V}(\theta)$ be the utility of individual θ when choosing the best alternative:

$$\overline{V}(\theta) = \arg\max[u_1(\theta),\ u_2(\theta), ...,\ u_n(\theta)] \tag{4}$$

The corresponding level of social welfare when each individual chooses his/her best alternative, $\overline{W}$, is

$$\overline{W} = \int \overline{V}(\theta) dF(\theta) \tag{5}$$

Denote by W_i the level of social welfare when *all* individuals choose alternative i:

$$W_i = \int u_i(\theta) dF(\theta) \tag{6}$$

We now state:

Proposition 1. (I) *If all alternatives do not yield the same utility for all individuals, then* $W(q)$ *strictly increases in* q; (II) $W(\infty) = \lim_{q\to\infty} W(q) = \overline{W}$ *and* (III) $W(0) = \lim_{q\to 0} W(q) = \frac{1}{n}\sum_{i=1}^{n} W_i$.

Proof. Using the definitions (1), (2) and (3), it can be shown that

$$\frac{dW(q)}{dq} = \int \left[\sum_{i=1}^{n} (u_i(\theta) - V(\theta))^2 p_i(\theta) \right] dF(\theta), \tag{7}$$

which, under the assumption, proves (I). Statements (II) and (III) also follow directly from the definitions (1), (2) and (3) ■.

Under *'perfect-rationality'*, $q = \infty$, each individual makes the 'right choice', and hence there is no reason, from a social welfare point of view, to limit the choice set. By continuity, this conclusion applies to large levels of q. When individuals make small errors, the full set of alternatives can best accommodate the diversity of individual preferences represented by θ. At the other extreme, when all alternatives have equal probabilities to be chosen by all individuals, $q = 0$, a large set of alternatives exacerbates ('spreads') the errors that individuals make to an extent that clearly outweighs the benefits of diversity. It is then socially optimal to drastically reduce the set of alternatives to a singleton, i.e. not to provide individuals any choice. This single socially preferred alternative depends, of course, on the distribution of population characteristics, accommodating as large as possible a mass concentration of individuals. The proof of this statement is straightforwards.

Let

$$W_m \subseteq \arg\max(W_1,\, W_2, ...,\, W_n) \tag{8}$$

In case of ties, the following applies to any element in the set (8).

We now have:

Proposition 2. *If* $W_i,\ i = 1,\, 2, ...,\, n$ *are not all equal, then when* $q = 0$ *social welfare is maximized by offering only alternative* m.

Proof. Follows directly from the fact that $W(0) = \frac{1}{n}\sum_{i=1}^{n} W_i$ and from (8) ■.

Since $W(q)$ is continuous in q, the implication of Proposition 2 is that there exists a positive level of q, say q_0, such that for all $q \leq q_0$, restricting the choice set confronting individuals to alternative m alone is socially desirable. As q increases above q_0, it is desirable to expand the set of alternatives, eventually including all of them for large q. It is interesting to inquire whether this 'expansion process' is gradual, i.e. whether it is optimal to include one or more alternatives simultaneously as q increases from q_0[6].

It can be shown that all these cases are possible. Here we shall focus only on q_0, characterizing the extent of errors that warrant the elimination of individual choice.

[6] Let T be a subset of $\{1, 2, , n\}$, standing for the indices of the alternatives offered to individuals. Consider the addition of alternative s, not included in T. Denote by $T' = sUT$, and let W^T and $W^{T}\prime$ be the corresponding levels of social welfare when the alternatives in $T(T')$ are offered to individuals. It can be shown that

$W^{T'} - W^T = \int p_s^{T'}(\theta)(u_s(\theta) - V^T(\theta)dF(\theta))$, where $p_s^{T'}(\theta)$ is the probability that individual θ chooses alternative s when confronting the set T' and $V^T(\theta)$ is expected utility of individual θ when confronting the set T. A necessary and sufficient condition for s to be included in the socially optimal set for some q is that, for some subset T, $u_s(\theta) - V^T(\theta) > 0$ in an interval of θ with a positive density. Sufficiency follows from the fact that as q increases, $p_s^{T'}(\theta)$ approaches zero for those θ for which $u_s(\theta) - V^T(\theta) < 0$ and approaches 1 on the complementary set of θ.

Starting with a single alternative at $q = q_0$, one can formulate an algorithm for inclusion of additional alternatives as q increases, so that welfare strictly increases with q at every step. It is easy to see that a discrete increase of $\Delta q > 0$ in q may lead to an inclusion of more than one additional alternative.

4 Self-Selection and Aggregate Constraints

The model outlined above has the property that, under perfect rationality ($q = \infty$), the economy attains the *First-Best* allocation of resources when individuals are offered the complete set of alternatives. At the other extreme, we concluded that when individual choice probabilities are uniformly distributed ($q = 0$), it is socially optimal to eliminate individual choice. Both conclusions have to be modified when the model incorporates private information not available to the government whose policies affect individual choice. The argument, however, that it is socially advantageous to reduce the choice set confronting individuals as they make increasing errors is still valid.

Suppose that the individual characteristic θ is private information, the government having only information on the distribution $F(\theta)$. Government policies affecting individual utilities and choice probabilities cannot therefore depend on θ. It is well-known that in these circumstances, even when individuals are perfectly rational, optimal policies lead to a *Second-Best* (*'Self-Selection'*) equilibrium. When individuals are boundedly rational, the socially optimal policies have to take into account their effect on the choice probabilities. This leads, in particular, to a modification of the conclusion that when probabilities are uniformly distributed ($q = 0$) it is always optimal to eliminate individual choice. This stark result may still be optimal, but other optimal configurations are possible. The reason is that, under an aggregate resource constraint, the government's optimal policies are different when a single alternative is offered than when a number of alternatives are permitted.

Let $u_i = u_i(x_i, \theta)$, $i = 1, 2, ..., n$, be individual θ's utility of alternative i, where x_i is some government policy (independent of θ). Choice probabilities, $p_i = p_i(x_i, \theta)$, and expected utility $\sum_{i=1}^{n} p_i(x_i, \theta)\, u_i(x_i, \theta)$ all depend on x_i. The government's objective is to choose $(x_1, x_2, ..., x_n)$ that maximize social welfare subject to a resource constraint

$$\int \left[\sum_{i=1}^{n} p_i(x_i,\, \theta)\, c_i(x_i) \right] dF(\theta) = R \tag{9}$$

where $c_i(x_i)$ is the cost of x_i in terms of the given level of the aggregate resource, R. Denote the policies that maximize W subject to the resource constraint (9) by $\widehat{x}_i(q)$, $i = 1, 2, ..., n$, and the corresponding level of social welfare by $\widehat{W}(q)$:

$$\widehat{W}(q) = \int \left[\sum_{i=1}^{n} p_i(\widehat{x}_i(q),\, \theta)\, u_i(\widehat{x}_i(q),\, \theta) \right] \tag{10}$$

A full-information policy would make x_i depend on θ and hence $\widehat{W}(\infty)$ is a *Second-Best* equilibrium.

When $q = 0$,

$$\widehat{W}(0) = \frac{1}{n} \sum_{i=1}^{n} W_i(\widehat{x}_i(0)) \tag{11}$$

where $W_i(\widehat{x}_i(0)) = \int u_i(\widehat{x}_i(0),\, \theta) dF(\theta)$, and $\widehat{x}_i(0)$ is the limit of $\widehat{x}_i(q)$ as $q \to 0$.

Let $\widetilde{x}_m$ be the feasible policy when only alternative m is permitted. By (9), it is the solution to $c_m(\widetilde{x}_m) = R$. The corresponding social welfare is $\widetilde{W}_m = \int u_m(\widetilde{x}_m) dF(\theta)$. It is seen that even if $W_m(\widehat{x}(0)) = \arg\max$ $[W_1(\widehat{x}_1(0)), W_2(\widehat{x}_2(0)), ..., W_n(\widehat{x}_n(0))]$, it does not follow necessarily that $\widetilde{W}_m > \widehat{W}(0)$[7]. Eliminating choice at $q \leq q_0$ is now only one of a number of possible outcomes.

Rather than develop sufficient conditions for the single outcome, we turn to a detailed example that incorporates the above considerations.

[7] Or that any other single alternative yield higher welfare than $\widehat{W}(0)$.

5 A Work-Retirement Model

Individuals can choose whether to work or retire. We take work to be a $0-1$ variable and do not model varying hours or work intensity. All workers are assumed to have the same (non-negative, increasing and concave) utility of consumption, $u(c)$, while all non-workers have (non-negative, increasing and concave) utility of consumption, $v(c)$[8]. Disutility of work, denoted θ, is assumed to be additively separable from consumption, so the utility of a worker with labor disutility level θ is written $u(c_a) - \theta$. The distribution of disutilities plays a central role in determining the willingness to work.

We consider two different consumption levels: c_a for active workers and c_b for non-workers ("retirement benefits"). We assume that θ is distributed in the population with distribution $F(\theta)$ (and density $f(\theta)$). For convenience, we assume that $f(\theta)$ is continuous and positive for all non-negative values of θ. Having individuals with a very large labor disutility implies, of course, that some will choose not to work.

Each worker is assumed to produce one unit of output. Thus, if all individuals with labor disutility less than some value, θ_0, work, the economy's resource constraint is

$$\int_0^{\theta_0} (c_a - 1)dF(\theta) + \int_{\theta_0}^{\infty} c_b dF(\theta) = R, \tag{12}$$

where R is the level of outside resources available to the economy. It is assumed, of course, that $R > -1$ to enable positive consumption.

5.1 First-Best (Labor Disutility Observable)

To set the stage for the optimal policy when labor disutility is not observable, it may be useful to analyze first the full optimum where disutility is observable.

[8] We also assume that both u' and v' go from $+\infty$ to c as c increases from 0 to $+\infty$.

The government's objective is to maximize a utilitarian social welfare function, W,

$$W = \int_0^{\theta_0} (u(c_a) - \theta) dF(\theta) + \int_{\theta_0}^{\infty} v(c_b) dF(\theta), \tag{13}$$

subject to the resource constraint (12). Optimal consumption, c_a^* and c_b^*, is allocated so as to equate the marginal utilities of consumption of workers and non-workers

$$u'(c_a^*) = v'(c_b^*) \tag{14}$$

All individuals with disutility below a cutoff, denoted θ^*, should work. The cutoff is determined by comparing the utility gain from extra work, $u(c_a^*) - \theta^* - u(c_b^*)$ with the value of extra net consumption as a consequence of work, which is the sum of the marginal product and the change in consumption which results from the change in status:

$$u(c_a^*) - \theta^* - v(c_b^*) = u'(c_a^*)(1 - c_a^* + c_b^*) \tag{15}$$

Whether the solution is interior, $\theta^* > 0$, depends on the wealth of the economy, i.e. the size of R. If no one works, optimal consumption is $c_b^* = R$. This allocation is not optimal if those with no labor disutility[9], when put to work for additional consumption equal to their marginal product, enjoy a higher utility

$$u(R + 1) > v(R) \tag{16}$$

[9]This assumes that $f(\theta) > 0$ at $\theta = 0$. Otherwise, a condition similar to (16) below has to be assumed with respect to those with the least (positive) labor disutility for which $f(\theta) > 0$.

We assume that (16), which Diamond and Sheshinski (1995) call the '*poverty condition*', is satisfied. This condition ensures that there is some work at the optimal allocation[10]. It can also be seen that it is always optimal for some individuals not to work[11].

5.2 Labor Disutility Unobserved

Assume now that individuals choose whether they prefer working to receiving retirement benefits, or *vice-versa.* Those with disutilities of labor below a threshold value which equates utilities of workers and non-workers will choose to work. The threshold value, $\widehat{\theta}$, satisfies

$$\widehat{\theta} = Max[0, u(c_a) - v(c_b)]. \tag{17}$$

The government's problem is now restated as selection of c_a and c_b so as to maximize (13) subject to (12), with θ_0 not an independent choice parameter but replaced by $\widehat{\theta}$, (17), which is a function of c_a and c_b.

Using the Lagrange multiplier λ and assuming an interior solution, we can state the first-order conditions as the resource constraint, (12), and the following two equations:

$$\begin{aligned}(u'(c_a) - \lambda) \int_0^{\widehat{\theta}} dF(\theta) &= \lambda(c_a - 1 - c_b) f(\widehat{\theta}) \tfrac{d\widehat{\theta}}{dc_a} \\ &= \lambda(c_a - 1 - c_b) f(\widehat{\theta}) u'(c_a)\end{aligned} \tag{18}$$

$$\begin{aligned}(v'(c_b) - \lambda) \int_{\widehat{\theta}}^{\infty} dF(\theta) &= \lambda(c_a - 1 - c_b) f(\widehat{\theta}) \tfrac{d\widehat{\theta}}{dc_b} \\ &= -\lambda(c_a - 1 - c_b) f(\widehat{\theta}) v'(c_b)\end{aligned} \tag{19}$$

[10] By (12), when $\theta^* = 0$, $c_b^* = R$. Setting $c_a^* = R + 1$, condition (16) implies that the L.H.S. of (15) is larger than the R.H.S. (= 0).

[11] If there are no non-workers then $c_b^* = 0$ and $c_a^* = R + 1$. It follows from (15) that the utility of the marginal worker is negative. Since $v(0) = 0$, this cannot be an equilibrium.

where we have used (17) to obtain the derivatives of $\widehat{\theta}$. The L.H.S. of (18) and (19) are the net social values of giving consumption to workers and non-workers, respectively, rather than holding the resources. The R.H.S. are the social values of the resource savings from the induced changes in labor supply as a result of changes in c_a and c_b, respectively. The private return to work is $c_a - c_b$ and hence, since the marginal product is one, $1 - c_a + c_b > 0$ is an implicit tax on work.

Denote by $(\widehat{c}_a, \widehat{c}_b)$ optimal consumption which solves (18) and (19) and denote by $\widehat{\lambda}$ the corresponding optimal Lagrangean. Dividing (18) by $u'(\widehat{c}_a)$ and (19) by $v'(\widehat{c}_b)$ and adding these equations, we get

$$\widehat{\lambda}^{-1} = (u'(\widehat{c}_a))^{-1} \int_0^{\widehat{\theta}} dF(\theta) + (v'(\widehat{c}_b))^{-1} \int_{\widehat{\theta}}^{\infty} dF(\theta) \tag{20}$$

That is, the inverse of the Lagrangian is a weighted average of the inverse of the marginal utilities at the optimum, $u'(\widehat{c}_a)$ and $v'(\widehat{c}_b)$.

We shall now impose a condition that will imply that, at the optimum, the marginal utility of consumption of non-workers exceeds that of workers. This is the Diamond-Mirrlees (1978) *moral-hazard condition*, that equating utilities between non-workers and workers who dislike work the least leaves marginal utility higher for non-workers:

$$u(x) = v(y) \text{ implies } u'(x) < v'(y). \tag{21}$$

Since $\widehat{\theta} > 0$, (17) implies that $u(\widehat{c}_a) > v(\widehat{c}_b)$. Hence,by (21), $u'(\widehat{c}_a) < v'(\widehat{c}_b)$. By (20) then, $u'(\widehat{c}_a) < \lambda < v'(\widehat{c}_b)$, and it follows from (18)-(19) that $1 - \widehat{c}_a + \widehat{c}_b > 0$, i.e. at the optimum there is a positive implicit tax on work.

6 Logit Model of Randomness in Individual Decisions

Instead of the previous deterministic model of individual choice, suppose now that an individual with labor disutility θ chooses to work with probability p_a, where

$$p_a = p_a(c_a, c_b, q, \theta) = \frac{e^{q(u(c_a)-\theta)}}{e^{q(u(c_a)-\theta)} + e^{qv(c_b)}}, \tag{22}$$

This is a special case of the logit model presented in Sections 2 and 3. It is seen that as q, the degree of accuracy ('rationality'), increases from 0 to $+\infty$, p_a increases (decreases) from $p_a = \frac{1}{2}$ for all θ to $p_a = 1(= 0)$ for $\theta < (>)\widehat{\theta} = u(c_a) - v(c_b) > 0$[12]. Figure 1 depicts p_a as a function of θ.

The sum of the probabilities of all individuals who should work, that is those with $\theta < \widehat{\theta} = u(c_a) - v(c_b)$, relative to this sum under full rationality $(= \widehat{\theta})$, can be viewed as an indicator for the effects of 'bounded-rationality'. Appendix A provides some calculations for representative values of c_a, c_b and alternative values of q. For the case $u(c) = v(c) = \ln c$, this proportion depends on the ratio c_b/c_a. With $c_b/c_a = .5$, it increases from .5 to .97 as q increases from 0 to 30. With $c_b/c_a = .7$ it reaches .94 and with $c_b/c_a = .9$ it reaches only .79, at $q = 30$. Thus, a higher c_b/c_a has a significantly negative effect on this proportion.

With choice of work and retirement governed by the probabilities p_a, social welfare is expressed as the sum of expected utilities:

$$W = \int_0^\infty [(u(c_a) - \theta)p_a + v(c_b)(1 - p_a)]\, dF(\theta). \tag{23}$$

[12]When $u(c_a) - v(c_b) < 0$, p_a decreases to 0 for all θ as q approaches $+\infty$.

Accordingly, the resource constraint is given by

$$\int_0^\infty \left[(c_a - 1)p_a + c_b(1 - p_a)\right] dF(\theta) = R. \tag{24}$$

Maximization of (23) with respect to c_a and c_b subject to (24), taking into account the dependence of p_a on these variables, (22), yields, in addition to (24), first-order conditions:

$$\begin{aligned}
&(u'(c_a) - \lambda) \int_0^\infty p_a dF(\theta) \\
&= \int_0^\infty \left[\lambda(c_a - 1 - c_b) - (u(c_a) - \theta - v(c_b))\right] \tfrac{\partial p_a}{\partial c_b} dF(\theta) \\
&= \int_0^\infty \left[\lambda(c_a - 1 - c_b) - (u(c_a) - \theta - v(c_b))\right] \\
&\quad p_a(1 - p_a) q u'(c_a) dF(\theta)
\end{aligned} \tag{25}$$

$$\begin{aligned}
&(v'(c_b) - \lambda) \int_0^\infty (1 - p_a) dF(\theta) \\
&= \int_0^\infty \left[\lambda(c_a - 1 - c_b) - (u(c_a) - \theta - v(c_b))\right] \tfrac{\partial p_a}{\partial c_b} dF(\theta) \\
&= -\int_0^\infty \left[\lambda(c_a - 1 - c_b) - (u(c_a) - \theta - v(c_b))\right]
\end{aligned}$$

$$p_a(1 - p_a) q v'(c_b) dF(\theta), \tag{26}$$

where we used that $\frac{\partial p_a}{\partial c_a} = q p_a (1 - p_a) u'(c_a)$ and $\frac{\partial p_a}{\partial c_b} = q p_a (1 - p_a) v'(c_b)$.

The interpretation of these conditions is basically unchanged: the L.H.S. of (25) and (26) is the expected net value of additional consumption to workers and non-workers, respectively. The R.H.S. is expected next resource cost of this additional consumption, which now includes the effect on all '*intra-marginal*' net utilities (i.e. those for which $u(c_a) - \theta - v(c_b)$ is positive or negative) of a change in the probability of work as a result of the change in consumption.

Denote the optimal consumption levels which solve (24), (25) and (26) by $(\widehat{c}_a(q), \widehat{c}_b(q))$. These levels imply, in turn, an optimal probability $\widehat{p}_a = \widehat{p}_a(q, \theta) = \widehat{p}_a(\widehat{c}_a, \widehat{c}_b, q, \theta)$ for an individual with labor disutility θ to choose to work. The corresponding optimal level of social welfare, denoted $\widehat{W}(q)$, is given by (23): $\widehat{W}(q) = \int_0^\infty [(u(\widehat{c}_a) - \theta)\widehat{p}_a + v(\widehat{c}_b)(1 - \widehat{p}_a)]dF(\theta)$.

We shall be interested in the dependence of these optimal values on the degree of rationality, q.

As q goes to $+\infty$, $\widehat{p}_a$ approaches 1 or 0 depending on whether θ is below or above $\widehat{\theta}$, (17), while $q\widehat{p}_a(1 - \widehat{p}_a)dF(\theta)$ approaches $f(\widehat{\theta})d\theta$ for $\theta = \widehat{\theta}$ and 0 elsewhere. Thus, (25)-(26) are a generalization of (18)-(19), converging to the latter as q goes to $+\infty$.

Dividing (25) and (26) by $u'(\widehat{c}_a)$ and $v'(\widehat{c}_b)$, respectively, and adding these equations we get:

$$\widehat{\lambda}^{-1} = (u'(\widehat{c}_a))^{-1} \int_0^\infty \widehat{p}_a dF(\theta) + (v'(\widehat{c}_b))^{-1} \int_0^\infty (1 - \widehat{p}_a) dF(\theta) \tag{27}$$

As before, the inverse of the optimal Lagrangean is a weighted average of the inverse of the marginal utilities.

7 Dependence of the Optimum on the Degree of Rationality

When individuals have a choice between work and retirement, the optimal level of social welfare, $\widehat{W}(q)$, increases with the degree of rationality, $\frac{d\widehat{W}}{dq} > 0$, provided we assume that, for all q, there is an implicit optimal tax on work: $1 - \widehat{c}_a + \widehat{c}_b > 0$ (Appendix B). At one extreme, when individual decisions are fully rational and deterministic, $q = +\infty$, it is always optimal for some individuals with low labor disutility to work (the '*poverty condition*', (16)) and for some with large labor disutility to retire (we have assumed that $f(\theta) > 0$ for all positive θ). At the other end, when $q = 0$, individuals' choice between work and retirement is independent of the levels of c_a and c_b, being $p_a = \frac{1}{2}$ for all θ. That is, half of all individuals choose not to work, including those with very low labor disutility. It seems possible, therefore, that with relatively low q, the elimination of the retirement option altogether may dominate all feasible positive combinations of c_a and c_b. Similarly, since half of all individuals choose to work, including those with very high labor disutility, it seems possible that the elimination of the work option may also dominate all feasible positive combinations of c_a and c_b.

We want to demonstrate that these are valid possibilities:

Proposition 3. *When $q = 0$ the optimal allocation has one of the following forms: (a) consumption levels of workers and of non-workers equate their marginal utilities, $u'(\widehat{c}_a) = v'(\widehat{c}_b)$, and $\widehat{c}_a + \widehat{c}_b = 2R + 1$; or (b) the retirement option is eliminated, setting $\widehat{c}_a = R + 1$; or (c) the work option is eliminated, setting $\widehat{c}_b = R$.*

Inspection of (23) and (24) shows that in the presence of work and retirement options, when $q = 0$ and $p_a = \frac{1}{2}$ for all c_a, c_b and θ, W is maximized when

$$u'(\widehat{c}_a(0)) = v'(\widehat{c}_b(0)) \text{ where } \widehat{c}_a(0) + \widehat{c}_b(0) = 2R + 1 \tag{28}$$

The corresponding welfare level, $\widehat{W}(0)$, is:

$$\widehat{W}(0) = \frac{1}{2}[u(\widehat{c}_a(0) + v(\widehat{c}_b(0))] - \frac{1}{2}E(\theta) \tag{29}$$

where $E(\theta) = \int\limits_0^\infty \theta dF(\theta)$ is expected ('average') labor disutility.

Without a retirement option, $\widehat{c}_a = R + 1$ and the corresponding level of welfare, denoted W_a, is

$$W_a = u(R+1) - E(\theta) \tag{30}$$

Without a work option, $\widehat{c}_b = R$ and the corresponding level of welfare, denoted W_b, is

$$W_b = v(R) \tag{31}$$

We want to show that each of (29), (30) or (31) is a possible optimum. For this purpose, it suffices to examine a special case. Thus, assume that $u(c) = v(c)$. Then, from (28), $\widehat{c}_a = \widehat{c}_b = R + \frac{1}{2}$ and $\widehat{W}(0) = u(R + \frac{1}{2}) - \frac{1}{2}E(\theta)$. Elimination of the retirement option is optimal iff $W_a > \widehat{W}(0)$. By (29) and (30), the following condition has to hold:

$$u(R+1) - E(\theta) > u(R + \frac{1}{2}) - \frac{1}{2}E(\theta) \tag{32}$$

or

$$u(R+1) - u(R + \frac{1}{2}) > \frac{1}{2}E(\theta). \tag{33}$$

By concavity, $u(R+1)-u(R+\frac{1}{2}) > \frac{1}{2}u'(R+1)$, hence a *sufficient* condition for (33) is

$$u'(R+1) > E(\theta). \tag{34}$$

By (30) and (31), $W_a > W_b$ is equivalent to $u(R+1) - u(R) > E(\theta)$. Hence, condition (32) is also sufficient for no-retirement to dominate the no-work program.

Similarly, from (29) and (31), $W_b > \widehat{W}(0)$, iff

$$u(R) > u(R + \frac{1}{2}) - \frac{1}{2}E(\theta) \tag{35}$$

or

$$u(R + \frac{1}{2}) - u(R) < \frac{1}{2}E(\theta) \tag{36}$$

Since $u(R + \frac{1}{2}) - u(R) < \frac{1}{2}u'(R)$, a *sufficient* condition for (36) is

$$u'(R) < E(\theta) \tag{37}$$

Again, (37) implies that $W_b > W_a$, that is, eliminating the work option is optimal. Obviously, (34) and (37) are mutually exclusive: either a no-retirement program or a no-work program are optimal.

Calculations presented in the next section indicate the following solution pattern. As the degree of rationality, q, decreases, so does the optimal level of welfare, $\widehat{W}(q)$. The gap between optimal consumption of workers and non-workers first increases in order to mitigate the rise in the probability of errors due to lower rationality. However, at still lower levels of q, the gap between workers' marginal utility of consumption and that of non-workers becomes more important than the maintenance of low probabilities of error and consequently the consumption gap starts to decrease, eventually disappearing (for $u(c) = v(c)$) when inevitable errors are unaffected by policy (at $q = 0$). Figure 2 presents this pattern which assumes that $\widehat{W}(0) > W_a > W_b$: preserving choice between work and retirement dominates a no-choice policy for all values of q. It is possible, however, as argued above, that elimination of the retirement option is dominant for small or moderate values of q ($W_a > \widehat{W}(0)$). This is depicted in Figure 3. Alternatively, it is possible that elimination of the work option becomes dominant for small or moderate q.

8 Logarithmic Two-Class Example

Consider the case where there are only two types in the economy, θ_1 and $\theta_2 (\theta_1 < \theta_2)$, with population weights f_1 and $f_2 = 1 - f_1$. Let workers and non-workers have the same logarithmic utility function: $u(c) = v(c) = \ln c$. Hence, workers and non-workers have equal consumption in the First-Best allocation. Accordingly, optimal social welfare when type one works and type two retires, W_1^* is

$$W^* = \ln(R + f_1) - \theta_1 f_1. \tag{38}$$

Social welfare when both types work, W_a, is

$$W_a = \ln(R + 1) - \theta_1 f_1 - \theta_2 f_2 \tag{39}$$

We shall assume that (38) is better than (39), which implies that

$$\theta_2 > \frac{1}{f_2} \ln \left(\frac{R+1}{R+f_1} \right) \tag{40}$$

Social welfare when nobody works, W_b, is

$$W_b = \ln R \tag{41}$$

We assumed that no work is inferior to type one working, $W^* > W_b$ (the '*poverty condition*', (16)) which holds iff

$$\theta_1 < \frac{1}{f_2} \ln \left(\frac{R+f_1}{R} \right) \tag{42}$$

(which is trivially satisfied when $\theta_1 = 0$[13]).

Under self-selection, the incentive compatibility condition for type one is $\ln(c_a) - \theta_1 \geq \ln c_b$. Since the marginal utility of non-workers is higher than that of workers, this condition holds with equality:

$$\ln c_a - \theta_1 = \ln c_b \quad \text{or} \quad c_a = c_b e^{\theta_1}. \tag{43}$$

Solving from the resource constraint

$$(c_a - 1) f_1 + c_b f_2 = R, \tag{44}$$

we have

$$\widehat{c}_b = \frac{R + f_1}{e^{\theta_1} f_1 + f_2}, \widehat{c}_a = \frac{e^{\theta_1}(R + f_1)}{e^{\theta_1} f_1 + f_2} \tag{45}$$

With this allocation, social welfare, $\widehat{W}(q = \infty)$, is

$$\widehat{W}(\infty) = \ln\left(\frac{R + f_1}{e^{\theta_1} f_1 + f_2}\right) \tag{46}$$

Comparing (46) and (49), the condition that at the self-selection optimum type two does not work is

$$\theta_2 > \frac{1}{f_2}\left[\ln\left(\frac{R+1}{R+f_1}\left(e^{\theta_1} f_1 + f_2\right)\right) - \theta_1 f_1\right] \tag{47}$$

Conditions (40) and (47) are the same when $\theta_1 = 0$, otherwise (46) is a somewhat more stringent condition than (40).

In the Logit model of Section 6, when $q = 0$, half the population works independent of the levels of consumption offered to workers and non-workers. Hence, it is optimal to offer both equal consumption: $\widehat{c}_a(0) = \widehat{c}_b(0) = R + \frac{1}{2}$.

As half the population works, social welfare, $\widehat{W}(0)$, is accordingly

$$\widehat{W}(0) = \ln(R + \frac{1}{2}) - \frac{1}{2}(\theta_1 f_1 + \theta_2 f_2) \tag{48}$$

[13]See footnote 9 above.

Without a retirement option, social welfare is given by (39). This level exceeds (48) iff

$$\theta_2 < \frac{1}{f_2}\left[2\ln\left(\frac{R+1}{R+\frac{1}{2}}\right) - \theta_1 f_1\right] \tag{49}$$

For (46) and (49) to be mutually satisfied it is required that

$$(R+1)(R+f_1) > (R+\frac{1}{2})^2(e^{\theta_1} f_1 + f_2) \tag{50}$$

For example, with $\theta_1 = 0$, condition (48) is $(R+1)f_1 > \frac{1}{4}$. This can clearly be satisfied. Thus, it is possible that a retirement alternative is desirable when individuals select their best alternative with certainty, but it is socially optimal to eliminate this alternative when individuals are less than perfectly rational.

Since $\widehat{W}(q)$ increases in q[14], whenever $W_a > \widehat{W}(0)$ there exists a 'cutoff' q such that $\widehat{W}(q)$ is larger (smaller) than W_a for q larger (smaller) than this value. It is interesting to examine at what level of q and the corresponding optimal solutions this cutoff occurs.

Optimal Values	q								
	0	1	2	4	7	10	20	100	∞
$\widehat{c}_a$	.50	.80	.88	.71	.62	.60	.57	.52	.50
$\widehat{c}_b$	.50	.32	.33	.32	.36	.38	.43	.48	.50
$\widehat{p}_a^1$	.50	.71	.88	.96	.98	1.00	1.00	1.00	1.00
$\widehat{p}_a^2$	.50	.36	.26	.06	.01	0	0	0	0
W	-1.07	-.99	-.88	-.80	-.75	-.72	-.71	-.69	-.69
W_a	-.75	-.75	-.75	-.75	-.75	-.75	-.75	-.75	-.75

Table 1[a]: $R = 0,\ \theta_1 = 0,\ \theta_2 = 1.5,\ f_1 = .5$

[14]a sufficient condition is a positive tax on work (see Appendix B).

Optimal Values	q								
	0	1	2	4	6	10	20	100	∞
$\widehat{c}_a$	.5	.97	1.00	1.00	.95	.87	.80	.71	.67
$\widehat{c}_b$	.5	.37	.43	.47	.50	.55	.60	.65	.67
$\widehat{p}_a^1$	.5	.72	.84	.95	.98	1.00	1.00	1.00	1.00
$\widehat{p}_a^2$	.5	.37	.21	.05	.01	0	0	0	0
$\widehat{W}$	-.94	-.80	-.65	-.52	-.50	-.44	-.42	-.41	-.41
W_a	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50

Table 2^a: $R = 0,\ \theta_1 = 0,\ \theta_2 = 1.5,\ f_1 = .67$

Optimal Values	q								
	0	1	1.5	2	10	20	40	100	∞
$\widehat{c}_a$	.5	.67	.71	.70	.60	.57	.54	.52	.50
$\widehat{c}_b$	.5	.27	.27	.27	.38	.43	.46	.48	.50
$\widehat{p}_a^1$	.5	.71	.81	.87	1.00	1.00	1.00	1.00	1.00
$\widehat{p}_a^2$	.5	.25	.18	.11	0	0	0	0	0
$\widehat{W}$	-1.19	-1.08	-1.00	-.93	-.73	-.71	-.70	-.69	-.69
W_a	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00

Table 3^a: $R = 0,\ \theta_1 = 0,\ \theta_2 = 2,\ f_1 = .5$

Optimal Values	q								
	0	1	2	4	10	20	40	100	∞
$\widehat{c}_a$	.5	.85	.98	.97	.86	.80	.74	.71	.67
$\widehat{c}_b$	.5	.34	.38	.45	.55	.60	.63	.65	.67
$\widehat{p}_a^1$	.5	.71	.87	.96	1.00	1.00	1.00	1.00	1.00
$\widehat{p}_a^2$	.5	.25	.11	.01	0	0	0	0	0
$\widehat{W}$	-1.03	-.85	-.67	-.52	-.44	-.42	-.41	-.41	-.41
W_a	-.67	-.67	-.67	-.67	-.67	-.67	-.67	-.67	-.67

Table 4^a: $R = 0,\ \theta_1 = 0,\ \theta_2 = 2,\ f_1 = .5$

For each parameter configuration: $\widehat{p}_a^i = \frac{1}{1+\left(\frac{\widehat{c}_b}{\widehat{c}_a}\right)^q e^{q\theta_i}}$, $i = 1, 2$ and $W_a = -\theta_2 f_2 = -\theta_2(1 - f_1)$.

Tables 1-4 present calculations of the optimal values $\widehat{c}_a, \widehat{c}_b, \widehat{p}_a^i = \widehat{p}_a(\widehat{c}_a, \widehat{c}_b, q, \theta_i)$, $i = 1, 2$, and $\widehat{W}(q)$ for given parameters $(R, \theta_1, \theta_2, f_1)$ and alternative values of q[15]. The parameters R and θ_1 are fixed at $R = \theta_1 = 0$, while θ_2 and f_1 are varied to examine their effect on the optimal solution. With $R = 0$, a no-work program without choice cannot be a possible optimum.

With these parameter values, (48) becomes $\widehat{W}(0) = -\ln 2 - \frac{1}{2}\theta_2 f_2$. By (46), $\widehat{W}(\infty) = \ln f_1$ and, by (39), $W_a = -\theta_2 f_2$. Thus, a unique cutoff between W_a and $\widehat{W}(q)$ occurs at a positive q iff $\theta_2 < \frac{2}{f_2}\ln 2$. The parameters in Tables 1-4 satisfy this condition.

In Tables 1 and 2 the cutoff occurs at q equal to 7 and 6, respectively. At these cutoff levels of q, the corresponding optimal ratios of non-worker to workers consumption, $\widehat{c}_b/\widehat{c}_a$, are around a half and the probabilities $\widehat{p}_a^1$ and $\widehat{p}_a^2$ are .98 and .01, respectively. The implication is that elimination of the retirement alternative is optimal when 98 percent or less of type 1 and 1 percent or more of type 2 individuals work, a rather surprising result. The change in the relative weight of the two types is seen to have only a marginal effect on the results.

Tables 3-4 take a higher value of type 2 labor disutility. As expected, the optimal elimination of the retirement option occurs now at lower levels of q, between 1.5 and 2 (compared to 7 and 6 previously). At these cutoff levels of q, the corresponding optimal consumption ratios, $\widehat{c}_b/\widehat{c}_a$, are significantly smaller: around a third. Accordingly, the optimal probabilities $\widehat{p}_a^1$ and $\widehat{p}_a^2$ at which the 'cutoff' occurs are now between .81 and .87 and between .18 and .11, respectively. Thus, elimination of the retirement option is optimal when around 85 percent or less of type 1 and 11 percent or more of type 2 individuals work. Again, the change in the relative weight of the two types has only a marginal effect.

[15] Calculations are based on MATLAB program for constrained maximization.

Comparing Tables 1-2 on the one hand and Tables 3-4 on the other, it is seen that, as expected, a larger spread of labor disutility in the population significantly reduces the levels of non-rationality for which it is optimal not to provide individuals any choice. Clearly, optimal choice provision is highly sensitive to population heterogeneity.

The most striking and surprising conclusion emerging from these calculations is *the wide range of solutions for which no choice is optimal.* Elimination of the retirement option compels all individuals to work. This includes a small proportion of individuals who, when having a retirement option, choose erroneously to retire and a large proportion of individuals with high labor disutility who choose not to work. This is a large cost which, presumably, outweighs the former benefit. There is, however, another benefit. The additional output produced by the entrants to the labor force enables a large increase in consumption to everyone. It is this benefit which tilts the outcome in favor of compulsion at very low levels of non-rationality. It is worth exploring further how general this finding is.

Appendix A

By (22),

$$p_a(\theta) = \frac{e^{q(u(c_a)-\theta)}}{e^{q(u(c_a)-\theta} + e^{qv(c_b)}} = \frac{1}{1 + ae^{q\theta}} \tag{1A}$$

where $a = e^{q[v(c_b)-u(c_a)]}$. The proportion of individuals who choose to work relative to those who should, denoted π, is

$$\pi = \frac{1}{\widehat{\theta}} \int_0^{\widehat{\theta}} p_a(\theta) d\theta = 1 - \frac{1}{q\widehat{\theta}} \ln \left[\frac{1 + ae^{q\widehat{\theta}}}{1 + a} \right] \tag{2A}$$

where $\widehat{\theta} = u(c_a) - v(c_b)$.

Let $u(c) = v(c) = \ln c$. Then, $a = \left(\frac{c_b}{c_a}\right)^q$ and $\widehat{\theta} = \ln\left(\frac{c_a}{c_b}\right)$. (2A) now becomes,

$$\pi = 1 - \frac{\ln 2 - \ln[1 + (c_b/c_a)^q]}{\ln(c_b/c_a)^q}. \tag{3A}$$

Assume that $c_b \leqslant c_a$. Using L'Hospital's rule, π is seen to approach 1 as q goes to $+\infty$ and to approach $\frac{1}{2}$ as q goes to 0. Table 1A presents select values of π for alternative values of q and $c_b/c_a = .5$, .7 and .9.

In all cases π increases rapidly from .5 when $q = 0$ to levels larger than .0 for values of q over 40. It is also seen that the rate of increase of π is significantly lower for higher values of c_b/c_a (which make no-work more attractive).

q	π $\frac{c_b}{c_a} = .5$	$\frac{c_b}{c_a} = .7$	$\frac{c_b}{c_a} = .9$
0	.50	.50	.50
1	.58	.54	.51
5	.81	.70	.57
10	.90	.81	.63
15	.93	.87	.68
20	.95	.90	.73
30	.97	.94	.79
40	.98	.95	.84
50	.98	.96	.87
70	.99	.97	.91
100	.99	.98	.93
∞	1.0	1.0	1.0

Table A.1

Appendix B

Given the definition of p_a, (22), $\frac{\partial p_a}{\partial c_a} = qp_a(1-p_a)u'(c_a)$, $\frac{\partial p_a}{\partial c_a} = -qp_a(1-p_a)v'(c_b)$ and $\frac{\partial p_a}{\partial q} = p_a(1-p_a)(u(c_a)-\theta-v(c_b))$. By (23), totally differentiating the optimum $(\widehat{c}_a, \widehat{c}_b)$ and $\widehat{p}_a$ w.r.t. q:

$$\begin{aligned}\frac{d\widehat{W}(q)}{dq} = & \int_0^\infty \{[\widehat{p}_a + (u(\widehat{c}_a) - \theta - v(\widehat{c}_b))q\widehat{p}_a(1-\widehat{p}_a)]u'(\widehat{c}_a)\tfrac{d\widehat{c}_a}{dq} + \\ & [(1-\widehat{p}_a) - (u(\widehat{c}_a) - \theta - v(\widehat{c}_b))q\widehat{p}_a(1-\widehat{p}_a)]v'(\widehat{c}_b)\tfrac{d\widehat{c}_b}{dq} + \\ & (u(\widehat{c}_a) - \theta - v(\widehat{c}_b))^2 p_a(1-p_a)\} dF(\theta)\end{aligned} \tag{B.1}$$

Differentiating totally the resource constraint, (24):

$$\begin{aligned}& \int_0^\infty \{[\widehat{p}_a + (\widehat{c}_a - 1 - \widehat{c}_b)q\widehat{p}_a(1-\widehat{p}_a)u'(\widehat{c}_a)]\tfrac{d\widehat{c}_a}{dq} + \\ & [(1-\widehat{p}_a) - (\widehat{c}_a - 1 - \widehat{c}_b)q\widehat{p}_a(1-\widehat{p}_a)v'(\widehat{c}_b)]\tfrac{d\widehat{c}_b}{dq} = \\ & \quad -\int_0^\infty \widehat{p}_a(1-\widehat{p}_a)(u(\widehat{c}_a) - \theta - v(\widehat{c}_b))\} dF(\theta)\end{aligned} \tag{B.2}$$

Inserting (B.2) and the F.O.C. (14)-(15) in the paper into (B.1), we obtain

$$\begin{aligned}\frac{d\widehat{W}(q)}{dq} = & \int_0^\infty \widehat{p}_a(1-\widehat{p}_a)(u(\widehat{c}_a) - \theta - v(\widehat{c}_b)) \\ & [u(\widehat{c}_a) - \theta - v(\widehat{c}_b) - \widehat{\lambda}(\widehat{c}_a - 1 - \widehat{c}_b)] dF(\theta).\end{aligned} \tag{B.3}$$

By (27), $\widehat{\lambda} > 0$. Thus, at an interior optimum, an implicit positive tax on work, $1 - \widehat{c}_a + \widehat{c}_b > 0$, implies $\dfrac{d\widehat{W}(q)}{dq} > 0$.

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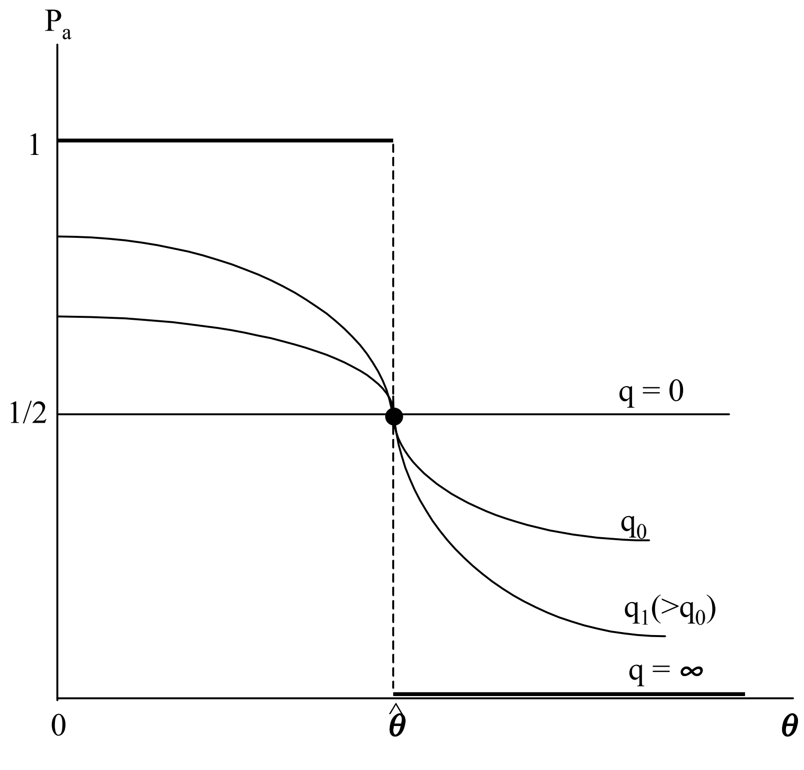
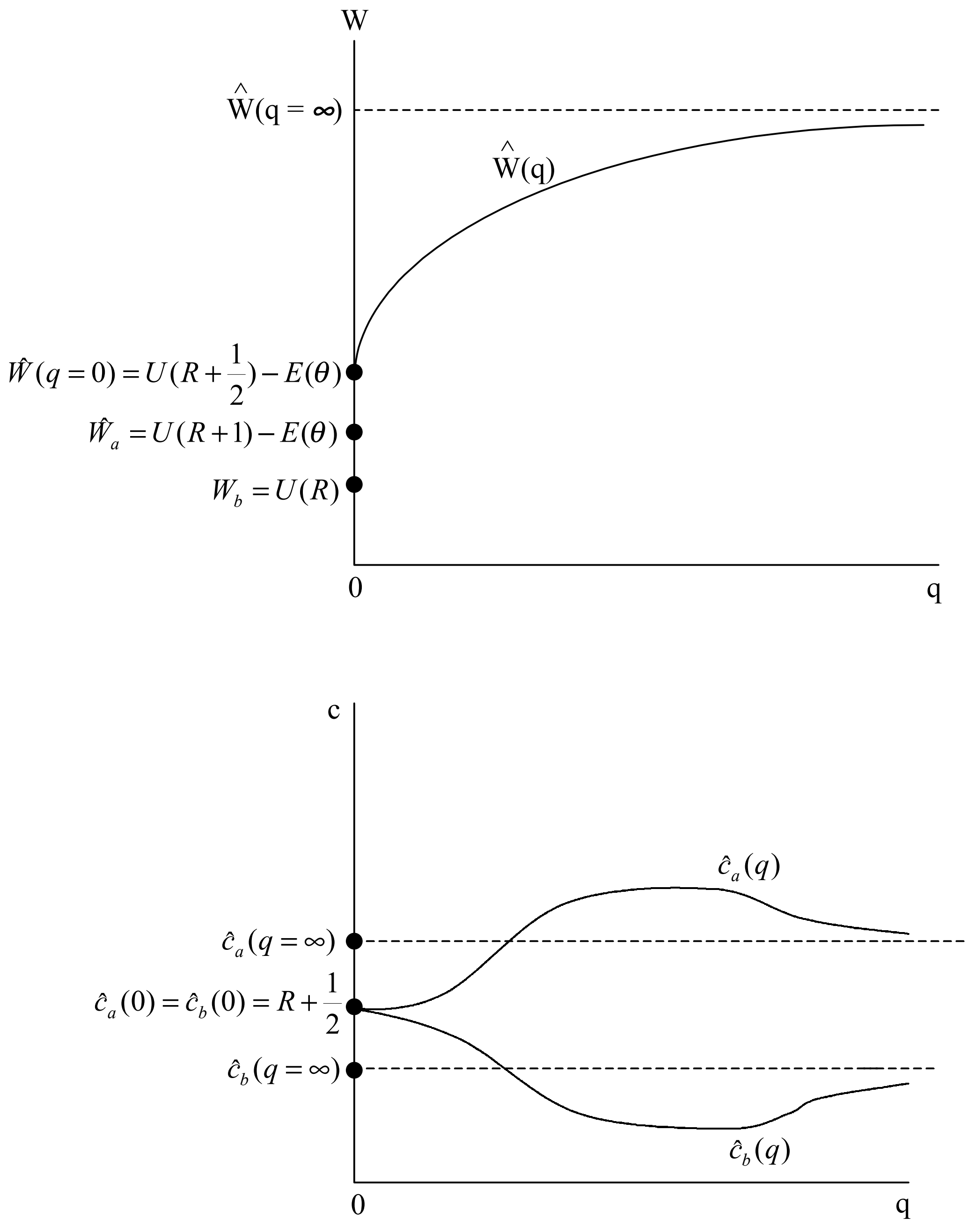
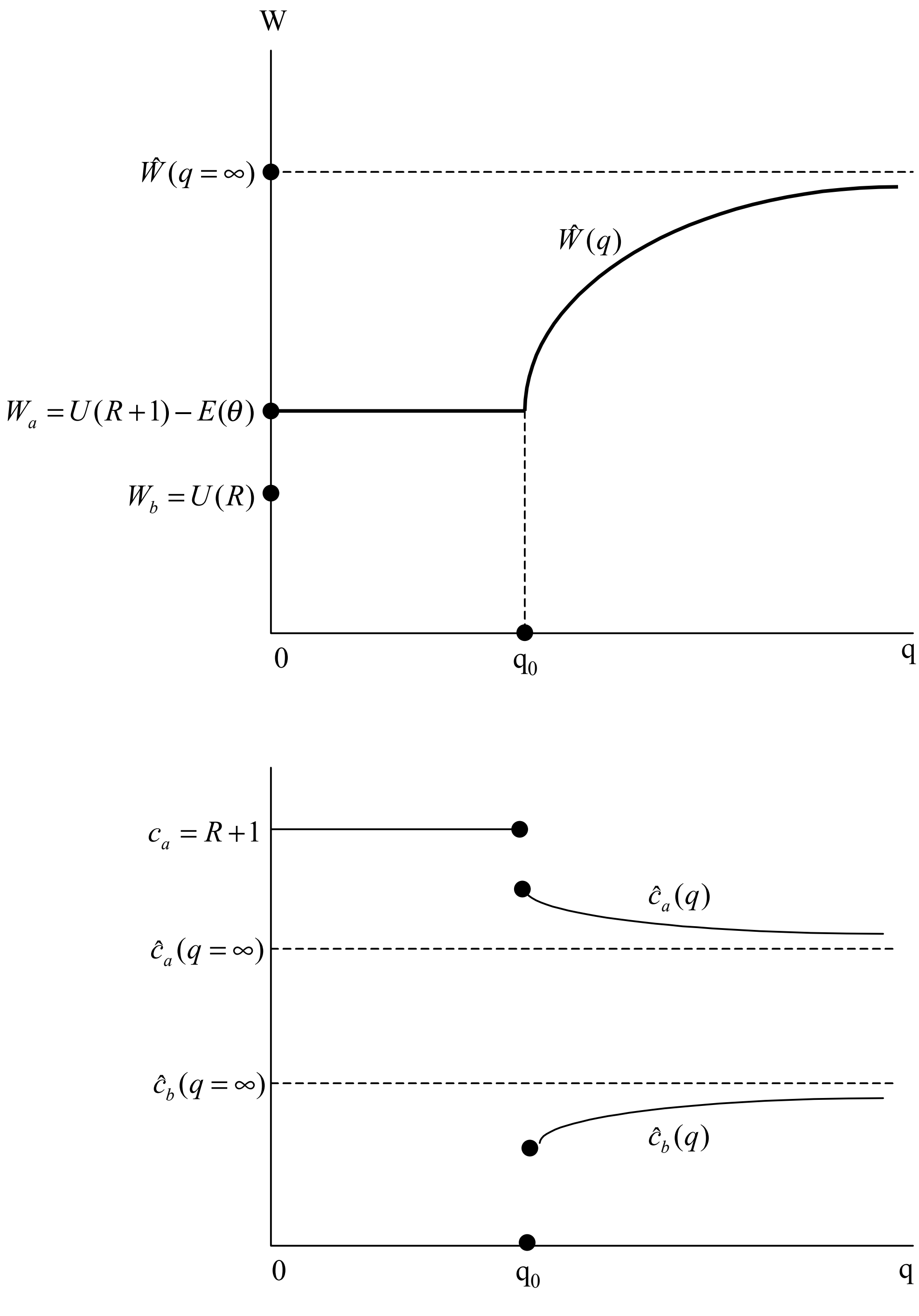


Figure 1



W
Ŵ(q = ∞)
Ŵ(q)
$\hat{W}(q=0)=U(R+\frac{1}{2})-E(\theta)$
$\hat{W}_a=U(R+1)-E(\theta)$
$W_b=U(R)$
0
q
c
$\hat{c}_a(q)$
$\hat{c}_a(q=\infty)$
$\hat{c}_a(0)=\hat{c}_b(0)=R+\frac{1}{2}$
$\hat{c}_b(q=\infty)$
$\hat{c}_b(q)$
0
q

Figure 2



W
$\hat{W}(q=\infty)$
$\hat{W}(q)$
$W_a = U(R+1) - E(\theta)$
$W_b = U(R)$
0
q_0
q
$c_a = R+1$
$\hat{c}_a(q)$
$\hat{c}_a(q=\infty)$
$\hat{c}_b(q=\infty)$
$\hat{c}_b(q)$
0
q_0
q

Figure 3