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OPTIMUM DELAYED RETIREMENT CREDIT

by

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Abstract

A central question for pension design is how benefits should vary with the age of retirement beyond early eligibility age. It is often argued that in order to be neutral with respect to individual retirement decisions benefits should be actuarially fair, that is, the present value of additional contributions and benefits ('Delayed Retirement Credit' - DRC) due to postponed retirement should be equal. We show that in a self-selection, asymmetric information model, because individual decisions are suboptimal, the socially optimal benefit structure should be less than actuarially fair.

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1 Introduction

A central question for pension design is how benefits should vary with the age of retirement beyond the earliest eligibility age.[1] For examples of widely varying pension benefits designs in many countries, see Gruber and Wise (1999). In the U.S., retirement ahead of the Normal Retirement Age (NRA), currently 65 but being raised to 67 by the year 2011, reduces benefits by 5/9 of one percent per month (about 6 percent annually). This is called the *'Actuarial Reduction Factor'*. Similarly, benefits increase for retirement beyond the NRA up to age 70. This is called *'Delayed Retirement Credit'* (DRC).

Workers vary in many ways - in life expectancy, income levels and in the degree of difficulty in continuing work. A good system needs to have flexibility in retirement ages to accommodate this diversity. (See Diamond (2000) lecture 3). It is often argued that it is desirable that the system be neutral with respect to individual retirement decisions, implying that the incentive design should be 'actuarially fair' on average. That is, the present value of additional contributions due to postponed retirement should equal the expected present discounted value of additional benefits.

The implicit assumption is that neutrality will preserve otherwise optimal individual decisions. We shall argue, however, that under asymmetric information, this is not the case. Certain individual attributes relevant to retirement decisions, such as labor disutility, are not observable by pension suppliers (government or private pension firms) and therefore pension schemes cannot depend on such attributes. Consequently, when individuals *self-select* their optimal retirement age based on their personal characteristics, the ensuing equilibrium is socially suboptimal: benefits to retirees are constrained by the need to provide sufficient incentives to continue work. DRC, by providing an incentive to continue to work, alleviates this constraint and leads to a better allocation of resources. This result holds even when all individuals have the same life expectancy (see Diamond (2000), lectures 6 - 7).

[1] Earliest elegibility age (62 in the U.S.) is designed to strike a balance between those who would, in the absence of such threshold, erroneously retire too early and others who have health or other reasons to retire earlier and for whom this imposes a liquidity constraint. One would like to know how this balance changes with increased life expectancy and morbidity. Interestingly, social security reforms in the U.S. and Sweden left the earliest elegibility age intact.

2 The Model

Consider three different consumption levels: c_a for active workers, c_b^0 for early retirees and c_b^1 for normal (or delayed) retirees. The utility function for a worker with labor disutility level θ is written $u(c_a) - \theta$. We assume that θ is non-negative and distributed in the population with distribution $F(\theta)$ (and density $f(\theta)$). For convenience, we assume that $f(\theta)$ is continuous and positive for all non-negative θ. The utility function of non-workers is $v(c)$, where c will take the values of c_b^0 or c_b^1, depending on the age at retirement.

We assume that the marginal product of workers is equal and normalize it to one. Thus, the only difference between workers is in the level of labor disutility.

Let T_0 be the age at which individuals have to make a decision whether to take early retirement or postpone retirement to age T_1, $T_1 > T_0$. With a certain life span of T, the length of retirement time is either $T - T_0$ for early retirees or $T - T_1$ for delayed retirement.

Normalizing the length of maximum retirement to one, delayed retirement entails work for a period of length α $(= \frac{T_1 - T_0}{T - T_0})$, $0 < \alpha < 1$, and retirement for a period of length $1 - \alpha (= \frac{T - T_1}{T - T_0})$, while early retirement is for a period of one. We further assume a zero subjective discount rate and zero rate of interest.[2]

If all those with labor disutility below a certain level, θ_0, work while the others take early retirement, social welfare, W, is

$$W = \alpha \int_0^{\theta_0} (u(c_a) - \theta) dF(\theta) + (1 - \alpha) \int_0^{\theta_0} v(c_a^1) dF(\theta) + \int_{\theta_0}^{\infty} v(c_b^0) dF(\theta). \quad (1)$$

[2]The results carry-over, with obvious changes, to the case with positive subjective discount and interest rates.

The budget constraint for the system is

$$\alpha \int_0^{\theta_0} (c_a - 1) dF(\theta) + (1-\alpha) \int_0^{\theta_0} c_b^1 dF(\theta) + \int_{\theta_0}^{\infty} c_b^0 dF(\theta) = R \tag{2}$$

where R are the resources available to the economy.

When $c_b^1 - c_b^0$ is positive, we call this difference the **Delayed Retirement Credit (DRC)**. Our objective is to analyze whether such credit is optimal and examine the dependence of its level on exogenous factors.

3 First-Best: Labor Disutility Observable

To ensure that the maximization of (1) s.t.(2) entails that some individuals work, we assume that when no one works, those with the least disutility of labor prefer to work for an additional consumption equal to their marginal product:

$$u(R + \alpha) > v(R) \tag{3}$$

This condition is called (Diamond-Sheshinski (1995)) the **poverty condition**.

When labor disutility is observable, it is possible to determine the optimum consumption levels and the cutoff θ so as to maximize (1) s.t.(2). Optimum consumption, $(c_a^*, c_b^{0*}, c_b^{1*})$, is allocated to equate marginal utilities of consumption:

$$u'(c_a^*) = v'(c_b^{0*}) = v'(c_b^{1*}). \tag{4}$$

All non-workers enjoy the same level of consumption, $c_b^{0*} = c_b^{1*}$. Consequently, the First-Best entails no DRC.

All individuals with disutility levels below a cutoff θ^*, $\theta^* > 0$, should work and the rest retire. The cutoff is determined by comparing the utility gain from extra work, $\alpha(u(c_a^*) - \theta^*) + (1-\alpha)v(c_b^{1*}) - v(c_b^{0*})$ with the value of extra consumption as a consequence of work, $u'(c_a^*)(c_a^* - \alpha - c_b^{0*})$.

By (4), this condition becomes:

$$\alpha(u(c_a^*) - \theta^* - v(c_b^{0*})) = u'(c_a^*)(c_a^* - \alpha - c_b^{0*}) \tag{5}$$

The *First-Best* allocation is determined by (4), (5) and the resource constraint (2).

4 Second-Best: Self-Selection Equilibrium

Suppose now that labor disutility is not observable. Consequently, the cutoff θ is determined by individuals: given consumption levels, those with disutility above the level which equates the utility of continued work and delayed retirement to that of early retirement, will prefer working. Thus, the threshold θ, $\widehat{\theta}$, is determined by:

$$\alpha(u(c_a) - \widehat{\theta}) + (1-\alpha)v(c_b^1) = v(c_b^0)$$

or

$$\widehat{\theta} = \frac{1}{\alpha}[\alpha u(c_a) + (1-\alpha)v(c_b^1) - v(c_b^0)]. \tag{6}$$

A sufficient condition to make a retirement program socially desirable is that the marginal utility of non-workers exceeds that of workers with the least disutility of work. This condition:

$$u(x) = v(y) \ \text{ implies } \ u'(x) < v'(y) \tag{7}$$

is termed (Diamond-Mirrlees, (1978)) the **moral hazard condition**. By (3), there is some work at the optimal allocation. Thus, at least the most able worker ($\theta = 0$) must work, implying $u(c_a) > v(c_b^0)$, and so, by (7), $u'(c_a) < v'(c_b^0)$.

Maximization of (1) s.t.(2), with θ_0 replaced by $\widehat{\theta}$ (which, by (6), is a function of c_a, c_b^0 and c_b^1) yields the following F.O.C.:

$$\alpha(u'(c_a) - \lambda) \int_0^{\widehat{\theta}} dF = \lambda A u'(c_a) \tag{8}$$

$$\alpha(v'(c_b^0) - \lambda) \int_{\widehat{\theta}}^{\infty} dF = -\lambda A v'(c_b^0) \tag{9}$$

$$\alpha(v'(c_b^1) - \lambda) \int_0^{\widehat{\theta}} dF = \lambda A v'(c_b^1) \tag{10}$$

where

$$A = [\alpha(c_a - 1) + (1 - \alpha)c_b^1 - c_b^0]\, f\,(\widehat{\theta}) \tag{11}$$

We have used (6) to obtain the derivatives of $\widehat{\theta}$ w.r.t. c_a, c_b^0 and c_b^1. The R.H.S. of these equations are the social values of resource savings from induced changes in labor supply due to altered benefits. The private return to working is $\alpha c_a + (1 - \alpha)c_b^1 - c_b^0$. Comparing this with the marginal product, α, we see that there is an implicit tax on work when $\alpha c_a + (1 - \alpha)c_b^1 - c_b^0 < \alpha$. As seen from (9), this is the case if at the optimum there are some non-workers and $v'(c_b^0) > \lambda$. When $\widehat{\theta}$, (6), is an interior solution then (7) ensures that this condition is satisfied.

Conditions (8)-(10) and the resource constraint (2) solve for optimum consumption and the corresponding Lagrangean, denoted $\widehat{c}_a$, $\widehat{c}_b^0$, $\widehat{c}_b^1$ and $\widehat{\lambda}$ respectively.

From (8) and (10) we see that $u'(\widehat{c}_a) = v'(\widehat{c}_b^1)$. Optimum delayed retirement benefits provide the same marginal utility as workers' consumption.[3]

Dividing (8)-(9) by the respective marginal utilities and adding, we see that the inverse of the Lagrangean equals the average of the inverses of the marginal utilities:

$$\widehat{\lambda}^{-1} = u'(\widehat{c}_a)^{-1} \int_0^{\widehat{\theta}} dF + v'(\widehat{c}_b^0)^{-1} \int_{\widehat{\theta}}^{\infty} dF. \tag{12}$$

Proposition. *When the optimum allocation has workers and non-workers, it has a positive delayed retirement credit (DRC) and an implicit tax on work.*

Proof. With $u'(\widehat{c}_a) = v'(\widehat{c}_b^1)$, we have from (6) and (7) that $u'(\widehat{c}_a) < v'(\widehat{c}_b^0)$. Hence, from (12), $u'(\widehat{c}_a) < \widehat{\lambda} < v'(\widehat{c}_b^0)$ and $v'(\widehat{c}_b^1) < v'(\widehat{c}_b^0)$ or $\widehat{c}_b^0 < \widehat{c}_b^1$.

The explanation of this result is straightforward. From the moral hazard condition, (6), we know that an attempt to implement the First-Best allocation, (4), is impossible because nobody will work. Any feasible policy that can increase the benefits of retirees without reducing workers' welfare is desirable. In the absence of DRC, $c_b^1 = c_b^0$, the cutoff $\widehat{\theta}$ is determined by the condition

[3] If $u(c)$ is a constant shift function of $v(c)$, then $\widehat{c}_a = \widehat{c}_b^1$.

$$\alpha(u(\widehat{c}_a) - \widehat{\theta}) + (1-\alpha)v(\widehat{c}_b^0) = v(\widehat{c}_b^0)$$

or

$$u(\widehat{c}_a) - \widehat{\theta} - v(\widehat{c}_b^0) = 0 \tag{13}$$

Now introduce a small DRC, raising retirement benefits for workers by $\dfrac{\Delta}{1-\alpha}$. Since these higher benefits apply for a period of $(1-\alpha)$, total costs for each worker increase by Δ and utility increases by $v'(c_b^0)\Delta$. Similarly, reducing workers' consumption by $\dfrac{\Delta}{\alpha}$ saves Δ over the working period α and decreases utility by $u'(c_a)\Delta$. By the moral hazard condition, $v'(\widehat{c}_b^0) - u'(c_a) > 0$, and hence workers' utility increases. Furthermore, the following inequality holds for the marginal worker,

$$\alpha(u(\widehat{c}_a) - \widehat{\theta}) + (1-\alpha)v(\widehat{c}_b^0) + (v'(\widehat{c}_b^0) - u'(\widehat{c}_a))\Delta > v(\widehat{c}_b^0), \tag{14}$$

implying that labor supply increases (by $f(\widehat{\theta})$). Since there is a tax on labor, $\widehat{c}_a - 1 - \widehat{c}_b^0 < 0$, this enables an increase in benefits for early retirees, c_b^0.

5 No Early Retirement

Suppose that θ has a finite upper bound, $\overline{\theta} > 0$. The *'poverty-condition'*, (3), ensures that the optimum involves some work. At the other end, suppose that the optimum allocation involves no non-workers, that is, $\widehat{\theta} = \overline{\theta}$. This means that the consumption of non-workers, $\widehat{c}_b^0$, is set at a level (possibly zero) such that nobody chooses early retirement. From (12) it follows that in this case, $\widehat{\lambda} = u'(\widehat{c}_a)$ $(= v'(\widehat{c}_b^1))$.

From (8) (or (10)) and (11)[4] we now obtain the condition, $\alpha(\widehat{c}_a - 1) + (1-\alpha)\widehat{c}_b^1 - \widehat{c}_b^0 \geq 0$. Since there are no non-workers to support, there is no

[4]Modified (Kuhn-Tucker) conditions for a boundary solution.

implicit tax on work[5].

6 Two-Class Case: Comparative Statics

Consider an economy with two types of individuals: those with labor disutility θ_1 and those with θ_2 $(\theta_1 < \theta_2)$. Population weights of these groups are f_1 and $f_2 = 1 - f_1$, respectively. We assume that the optimum has the form that the θ_1 types work while the θ_2 types take early retirement.

Social welfare optimization now takes the form

$$Max\left\{\left[\alpha(u(c_a) - \theta_1) + (1-\alpha)v(c_b^1)\right] f_1 + v(c_b^0)(1-f_1)\right\} \tag{15}$$

subject to

$$\left[\alpha(c_a - 1) + (1-\alpha)c_b^1\right] f_1 + c_b^0(1-f_1) = R \tag{16}$$

$$\alpha(u(c_a) - \theta_1) + (1-\alpha)v(c_b^1) \geq v(c_b^0) \geq \alpha(u(c_a) - \theta_2) + (1-\alpha)v(c_b^1) \tag{17}$$

Condition (17) ensures that individual behavior coincides with that described in the objective function and the resource constraint, (16). The **moral hazard condition** and (17) imply that

$$\alpha(u(c_a) - \theta_1) + (1-\alpha)v(c_b^1) = v(c_b^0) \tag{18}$$

[5] Another way to see this: from the resource constraint (2), when $\widehat{\theta} = \overline{\theta}$, $\alpha(\widehat{c}_a - 1) + (1-\alpha)\widehat{c}_b^1 = R$. Hence, $\widehat{c}_b^0 \leq R$ (in particular, with no exogenous resources, $R = 0$, $\widehat{c}_b^0 = 0$). None of the output of workers, α, is allocated to non-workers.

Performing the maximization of (15) subject to (16)-(18), we obtain that at the optimum $u'(\widehat{c}_a) = v'(\widehat{c}_b^1) < v'(\widehat{c}_b^0)$. Hence, $\widehat{c}_b^1 - \widehat{c}_b^0 > 0$.

We can use this example to analyze the effect on the optimal configuration of an increase in longevity, i.e. a decrease in α.

Differentiating (16) and (18) totally w.r.t. α, viewing $\widehat{c}_b^1$ as dependent on $\widehat{c}_a$ through the relation $u'(\widehat{c}_a) = v'(\widehat{c}_b^1)$, yields

$$\frac{d\widehat{c}_b^1}{d\alpha} = \frac{-\beta}{u'(\widehat{c}_a)(1-f) + v'(\widehat{c}_b^0)\, f_1} \left[(u(\widehat{c}_a) - \theta_1 - v(\widehat{c}_b^1))(1-f_1) + \right. \\ \left. + (\widehat{c}_a - 1 - \widehat{c}_b^1) v'(\widehat{c}_b^0)\, f_1 \right] \tag{19}$$

$$\frac{d\widehat{c}_b^0}{d\alpha} = \frac{-1}{u'(\widehat{c}_a)(1-f_1) + v'(\widehat{c}_b^0)\, f_1} \left[(\widehat{c}_a - 1 - \widehat{c}_b^1) u'(\widehat{c}_a) - \right. \\ \left. - (u(\widehat{c}_a) - \theta_1 - v(\widehat{c}_b^1)) \right] f_1 \tag{20}$$

where

$$\beta = \frac{u''(\widehat{c}_a)}{\alpha v''(\widehat{c}_b^1) + (1-\alpha) u''(\widehat{c}_a)} > 0.$$

From (12) and (18), $u(\widehat{c}_a) - \theta_1 - v(\widehat{c}_b^1) = \frac{1}{\alpha}(v(\widehat{c}_b^0) - v(\widehat{c}_b^1)) < 0$. Also, from (8) - (12), $\widehat{c}_a - 1 - \widehat{c}_b^1 < \frac{1}{\alpha}(\widehat{c}_b^0 - \widehat{c}_b^1) < 0$. It follows from (19) that $\frac{d\widehat{c}_b^1}{d\alpha} > 0$, while the sign of $\frac{d\widehat{c}_b^0}{d\alpha}$ is ambiguous.

By (19) - (20), the effect on the DRC is

$$\frac{d\widehat{c}_b^1}{d\alpha} - \frac{d\widehat{c}_b^0}{d\alpha} = \frac{-1}{u'(\widehat{c}_a)(1-f_1) + v'(\widehat{c}_b^0)\, f_1} [\beta(u(\widehat{c}_a) - \theta_1 - v(\widehat{c}_b^1)) + \\ + (1-\beta)(u(\widehat{c}_a) - \theta_1 - v(\widehat{c}_b^1)) f_1 + \\ + (\widehat{c}_a - 1 - \widehat{c}_b^1)(v'(\widehat{c}_b^0) - u'(\widehat{c}_a)) f_1] \tag{21}$$

A sufficient condition for (21) to be positive is that $\beta \leq 1$. This condition holds when $v''(\widehat{c}_b^1) \leq u''(\widehat{c}_a)$ or, in terms of coefficients of risk aversion (since $v'(\widehat{c}_b^1) = u'(\widehat{c}_a)$), $\frac{v''(\widehat{c}_b^1)}{v'(\widehat{c}_b^1)} \leq \frac{u''(\widehat{c}_a)}{u'(\widehat{c}_a)}$[6]. A reduction in α reduces total output and consequently the consumption of workers.

The change in consumption of non-workers depends on the magnitude of the change in output relative to the change in consumption of workers. The above condition implies that in order to maintain equal marginal utility of workers before and after their (delayed) retirement, the reduction in workers' retirement consumption, $\widehat{c}_b^1$, is not larger than the reduction in their consumption while working, $\widehat{c}_a$.

In the special case where $u(c) = v(c)$, since $\widehat{c}_a = \widehat{c}_b^1$, equations (19) - (21) take the simple form:

$$\begin{aligned}
\frac{d\widehat{c}_b^1}{d\alpha} &= \frac{\theta_1(1-f_1) + v'(\widehat{c}_b^0)\, f_1}{u'(\widehat{c}_a)(1-f_1) + v'(\widehat{c}_b^0)\, f_1} > 0 \\
\frac{d\widehat{c}_b^0}{d\alpha} &= \frac{u'(\widehat{c}_a) - \theta_1}{u'(\widehat{c}_a)(1-f_1) + v'(\widehat{c}_b^0)\, f_1} \gtreqless 0 \qquad (22) \\
\frac{d\widehat{c}_b^1}{d\alpha} - \frac{d\widehat{c}_b^0}{d\alpha} &= \frac{(v'(\widehat{c}_b^0) - u'(\widehat{c}_a))\, f_1 + \theta_1}{u'(\widehat{c}_a)(1-f_1) + v'(\widehat{c}_b^0)\, f_1} > 0
\end{aligned}$$

The sign of the last expression is unambiguous due to the **moral hazard condition**.

[6] This holds, for example, when $u(c)$ is a constant shift of $v(c)$.

Referencs

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