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Simple Subscription Mechanisms for Excludable Public Goods*

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For excludable public goods, we propose simple mechanisms to uniquely implement a (core) stable and efficient production and cost-sharing outcome: consumers are asked to announce sequentially their minimal requested level of public good and a subscription towards its production. In one mechanism the subscriptions depend on the order of moves. In a second mechanism, the subscriptions are order-independent and thus symmetric. The equilibrium outcomes induced by our mechanisms are immune to strategic deviations by coalitions. *Journal of Economic Literature* Classification Numbers: H41, C72, D78.

1. INTRODUCTION

Three key issues in the literature on public good provision are *stability*, *efficiency*, and *equity/fairness*: what level of production is economically best for the society is an efficiency question; whether a public good production and cost-sharing rule agreed to be obeyed by noncooperative game playing agents will result in a socially *just* outcome is the important issue

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of equity; finally, the equilibrium outcome(s) of the public good provision game must be *core* stable, i.e., immune to deviations by coalitions. To align all three objectives simultaneously is a difficult task.[1,2] In this paper we propose *simple* mechanisms that achieve these three objectives in the case of *excludable* public goods.[3]

Designing mechanisms/games to ensure a desirable public good production outcome has been a popular research theme. Earlier Bagnoli and Lipman [4] studied the core implementation problem in a discrete but non-excludable (i.e., pure) public good economy using undominated perfect equilibrium as the solution concept. More recently Moldovanu [11] has analyzed an infinite horizon bargaining game to show that negotiations via proposals and counter proposals can implement the core outcomes in stationary subgame perfect equilibria for excludable public goods. Moore and Repullo [13] even constructed mechanisms for general economic environments that under certain conditions allow implementation (in subgame perfect equilibrium) of a general social choice function for a public good provision problem. Our objective here departs from the above literature both in the choice of the implemented solution as well as in the nature of the implementing mechanism. Concerning the first, we are not interested in core implementation per se. In fact, core has been shown to be rather large in the case of public good production (see Moulin [14, p. 283]). Alternatively we aim at a unique outcome for the public good production problem, that in addition to being in the core also satisfies a *symmetry* property known in the literature as *horizontal equity*. Regarding the mechanism, our main concern is with *simplicity*. This turn on the issue of what constitutes a "good" mechanism. We believe a dominant criterion in this respect should be the prospects of actually applying the mechanism to real life problems that fall within the boundaries of the underlying model. This would rule out various abstract and complex mechanisms. In the mechanisms we propose, players act sequentially and each player gets to play not more than twice. Also, players' actions (i.e., messages) are very simple. They are requested to announce their desirable level of public good and a subscription towards the cost of its production. Thus our mechanisms do not deviate much from the way real life decisions about

[1] As discussed in Moulin [14, pp. 278–279], though the equal cost-sharing rule satisfies *no-envy*, an important criterion of fairness, it violates agent participation constraint and hence cannot induce a core outcome.

[2] Jackson and Moulin [9] address two of the issues—efficiency and equity; however, they consider only the indivisible public good case and thus the issue of provision of the efficient level is not fully explored.

[3] An excludable public good's consumption, though non-rival in nature, can be restricted to specific agents while excluding others at zero cost. Brito and Oakland [6] and Dreze [8] were some of the initial papers to study excludable public goods.

public good production are made, and therefore are more likely to be adopted than the abstract mechanisms available in the general implementation literature.[4]

We present two *sequential* mechanisms. Both mechanisms are based on sequential bids in which consumers announce their request for a minimal level of the public good and a subscription towards its production. This is related to the idea of demand commitment discussed by Selten [20] and later by Winter [23, 24] in the context of bargaining. Given the consumers' bids, the planner determines the level of public good by referring to the maximal coalition for which the maximal demand and the total subscription are compatible with constraints imposed by the production technology. In the first mechanism, the bids are announced with respect to a pre-specified order. The mechanism implements a unique, core (and thus efficient) outcome but one which is lopsided as agents moving earlier have a clear advantage. The second mechanism, which is somewhat like a twofold repetition of the first mechanism, corrects for the asymmetry by having the order of moves in the second period chosen randomly according to a uniform distribution. Thus the second mechanism not only achieves the efficient level of public good and yields a unique and core outcome, but also implements a symmetric outcome. Finally, our mechanisms have an additional desirable property: the induced equilibria are immune to strategic deviations by coalitions, i.e., they are (perfect) strong equilibria *à la* Aumann [3].

In view of the much familiar difficulties of free-riding/inefficiency (Bergstrom *et al.* [5], Andreoni [2], Admati and Perry [1], among others) and/or multiplicity of equilibria (Palfrey and Rosenthal [18], Bagnoli and Lipman [4], Varian [22]) even in simple public good environments with complete information, it is worth reviewing how the mechanisms suggested in this paper resolve the issues.[5] While we maintain the complete information assumption, our main departure from traditional models is in the excludable nature of the public good, because of which the agents promising to subscribe to the good's production can credibly threaten to exclude fellow potential users from its consumption unless all agents coordinate on cost-sharing, thus reducing the free-riding tendencies

[4] The complexity of most mechanisms in the general implementation literature is due to the large domain of their applications; for example, the mechanism in Moore and Repullo [13] applies to a wide class of social choice rules.

[5] A recent paper by Marx and Matthews [10] examines cumulative voluntary contribution decisions by agents to develop a public project, when some intermediate benefits can be derived even before the project's final completion. Under complete but imperfect information, free-riding is shown to vanish in the "limiting" equilibria (i.e., if the number of periods is large, discounting is low, and period length are small), although other inefficient equilibria do exist.

at individual as well as group levels. However, the dual objectives of efficiency and uniqueness, plus a third objective of immunity to coalitional deviations, cannot be achieved without the sequential structure of the demand-subscription postings. In voluntary contributions, sequentiality, by way of sunk contributions, creates more free-riding opportunities relative to simultaneous contributions; in contrast, the *conditional* commitment power conferred by our demand-subscription mechanisms, distinct from direct contribution games, is enhanced if the announcements are sequential rather than simultaneous, thus ruling out potential inefficient outcomes possible under simultaneous play;[6] finally, because simultaneous play leads to inefficient outcomes which are outside the core, there always remains the possibility that a group of players will all benefit by deviating from, and thus destabilizing, the equilibrium play, which is avoided in sequential mechanisms.

The rest of the paper is organized as follows. In Section 2, we describe the excludable public good model and define the stand alone core. The demand-subscription mechanisms and related results are presented in Section 3. Section 4 concludes. The proofs of two lemmas and one of the examples are included in the Appendix.

2. THE EXCLUDABLE PUBLIC GOOD MODEL

A public good (or facility) benefiting a group of agents, $N = \{1, 2..., n\}$, is called *excludable* if any agent's consumption can be restricted at zero cost to any level below the total quantity produced. Examples range from highways, airports, zoos, museums, cable TVs, etc.—as long as the facilities remain uncongested (see Brito and Oakland [6] on this)—to satellites serving multiple long distance telephone companies/TV channels, database shared by a network of college libraries, different community clubs/organisations with a varying range of services, etc. As Moulin [14, chap. 5, pp. 277–278] discusses, some public goods (see the first set of examples above) can be produced using only a *single copy* of the technology, and others (viz. the second category of examples) with a *free access* technology.[7,8] If it is a single copy technology, exclusion of an agent from

[6] It is the conditional aspect of commitment, not just commitment, that tackles the free-rider problems.

[7] Strictly speaking, Moulin's classification of technologies was for (pure) public goods, though the same classification also applies to excludable public goods.

[8] To be precise, Moulin uses the term *common property* technology to describe what we call single copy technology. The terminology we use is perhaps more appropriate in an excludable public good setting because an agent may after all be denied the use of the technology by a central authority with vested power.

the use of the overall supply of an (excludable) public good by some central authority/planner leaves the agent with "zero" reservation utility. On the other hand, if it is a free access technology, different agents can produce different levels of output $y_1, ..., y_n$, though, by non-rivalry, everyone can enjoy $y^+ = \max\{y_1, ..., y_n\}$ should exclusion not occur; again the planner can monitor consumption by the method of exclusion, but because an agent may now choose to produce and consume on his own, his or her stand alone payoff (i.e., reservation utility level) can be positive. For either technology the feasible allocations in a one input–one output economy are given by

$$\langle y; x_1, ..., x_n \rangle \qquad \text{such that} \quad y = f\left(\sum_{i=1}^{n} x_i\right), \quad f(0) = 0$$

and

$$0 \leqslant x_i \leqslant a_i \qquad \text{for all } i, \tag{1}$$

where y is the level of public good produced using technology $f(\cdot)$ and a_i is agent i's endowment of input.

For any level y of the public good, the benefit to agent i is denoted by $u_i(y)$ and the overall cost of production is denoted by $c(y)$, i.e., $c(\cdot)$ is the inverse of the production function $f(\cdot)$. To formally define core, we make the following specific assumption on the type of technology.

Assumption 1. Agents have a free access to the production technology $f(\cdot)$.

Let y^* be the efficient level of public good, i.e., y^* maximizes $\{\sum_{i=1}^{n} u_i(y) - c(y)\}$. The following assumptions ensure that the efficient level y^* is positive, bounded and unique.

Assumption 2. (a) The production technology $f(\cdot)$ is differentiable, increasing, and $f(0) = 0$. Equivalently, the cost function $c(\cdot)$ is differentiable and increasing in y, and $c(0) = 0$.

(b) The net utility to agent i, who subscribes x_i in input and consumes public good level y, is denoted by $u_i(y, x_i)$, which is differentiable and increasing in y and quasi-linear in x_i, i.e., $u_i(y, x_i) = u_i(y) - x_i$, and by normalization $u_i(0) = 0$.[9]

(c) $\sum_i u_i'(y)$ is decreasing in y, $\sum_i u_i'(y) \to 0$ as $y \to \infty$, and $\sum_i u_i'(0) > c'(0)$.

[9] The assumption of quasi-linearity is standard in the related literature on public goods. It allows us to represent the public good game as a transferable utility game.

(d) The downward sloping aggregate marginal benefit function, $\sum_i u_i'(y)$, cuts the marginal cost function, $c'(y)$, that may be increasing, constant or even decreasing, from above only once.[10]

Given our free access technology assumption, an appropriate concept of core is that of stand alone core defined by Moulin [14, p. 280] as follows:[11]

DEFINITION 1. Given is a public good provision problem $(u_1, ..., u_n; c)$. If $\langle y; x_1, ..., x_n \rangle$ is a feasible allocation given by (1), we say that coalition $S \subset N$ has a stand alone objection against it if there exists an S-allocation $\langle y'; x_j', j \in S \rangle$ such that

(i) $\sum_S x_j' = c(y')$, $0 \leqslant x_j' \leqslant a_j$, all $j \in S$;

(ii) $u_j(y', x_j') \geqslant u_j(y, x_j)$ for all $j \in S$, with at least one strict inequality.

The stand alone core is the set of feasible allocations against which no stand alone objection exists.

Under fairly mild assumptions on technology and preferences, stand alone core of the public good provision problem $(u_1, ..., u_n; c)$ is nonempty and quite large (see Moulin [14, Theorem 5.2(a)]); in particular, our assumptions 1 and 2 satisfy Moulin's requirements.[12] So any attempt to guarantee a (unique) core outcome by designing a game to be played by the agents noncooperatively is not a vacuous exercise.

3. DEMAND-SUBSCRIPTION MECHANISMS AND RESULTS

As in the general implementation literature we assume that the planner does not know anything about agents' preferences or endowment, but knows the production technology. The agents have complete information about each others' preferences and endowments plus the technology. We need a definition before we describe the mechanisms.

[10] Thus the production technology is quite general and accommodates all three types—decreasing, increasing, and constant returns to scale.

[11] With free access to the technology stand alone core has a positive interpretation, and in the single copy technology case the interpretation is normative. So we adopt the positive interpretation. Thus our framework differs from Moldovanu's [11] where all agents outside the final coalition receive the same payoff (i.e., zero utility) instead of different agents receiving different stand alone utilities: Moldovanu studied the single copy technology case.

[12] From here onwards the word, core, should always imply stand alone core.

DEFINITION 2. A (nontrivial[13]) coalition S is said to be *compatible* with respect to the announcements $(y_i, x_i)_{i \in N}$ if $f(\sum_{j \in S} x_j) \geqslant \max_{j \in S} y_j$. Coalition S is said to be a *maximal* compatible coalition if there is no strict superset of S which is compatible.

Note that because the union of every two compatible coalitions is itself compatible, the maximal compatible coalition, if it exists, is unique. We slightly abuse the notation S also to denote the maximal compatible coalition. We start with the following auxiliary mechanism.

The Mechanism Γ_1. At the beginning of the game the planner chooses an (exogenous) order of moves for the agents. Each agent i takes his turn to submit publicly a pair (y_i, x_i), which should be understood as a demand for at least y_i level of public good and a commitment to subscribe x_i towards the production. This concludes the agents' part in the mechanism.

Given the announcements $(y_i, x_i)_{i \in N}$, the planner determines the maximal compatible coalition S. Then he sets the level of public good at $f(\sum_{j \in S} x_j)$ and excludes all agents outside S. The excluded agents enjoy their respective stand alone utilities. This completes the description of the mechanism Γ_1.

An implication of the extensive form mechanism Γ_1 is the following Proposition.

PROPOSITION 1. *Let $E = (N, u, a; c(\cdot))$ be an (excludable) public good economy satisfying assumptions* 1 *and* 2. *For any chosen order w, the unique (subgame perfect) equilibrium outcome of the game, Γ_1, leads to no exclusion. The implemented level of public good is the efficient level y^* and the resulting allocation of subscriptions is stable (in the sense of the core).*

Since the proof of Proposition 1 is based on an intermediate result (Proposition 2), we postpone the proof temporarily.

Although stable and efficient, the (unique) equilibrium outcome of the mechanism Γ_1 depends on the exogenous order of moves chosen by the planner, and thus does not treat agents equally. So we now turn to a second mechanism, Γ_2, whose (unique) equilibrium outcome possesses a symmetry property in addition to the core stability.

DEFINITION 3. A public good production and cost-sharing outcome is said to be *symmetric* if it induces the same (net) utility for any two agents with identical preferences. This implies in particular that agents with the same preferences must make equal subscriptions.

The symmetry property—"equal treatment of equals"—is also known as *horizontal equity* (Musgrave [17, p. 160], Pazner and Schmeidler [19]).

[13] A nontrivial coalition must contain at least two agents. By coalition in this paper, we mean nontrivial coalition.

The Mechanism Γ_2. The mechanism Γ_2 is somewhat like a two-stage version of Γ_1. Specifically, the game starts by playing Γ_1 with respect to a predetermined order. After all agents make their announcements, they are asked sequentially whether they would like to replay the game. If all say NO, then the game ends with the planner determining the outcome according to the rule specified in Γ_1. If at least one player veto the outcome, then the planner samples a new order with a probability of $1/(n!)$ for each order and Γ_1 is played again with respect to the realized order. The resulting outcome from this round stands as the final outcome.

The intuition for the mechanism Γ_2 is as follows. That an agent can veto the first-round play in the game Γ_2, works as the disciplining device preventing early movers from taking "unfair" advantage. The use of uniform distribution to determine the order of the second-round play determines the agents' expected payoffs, which, in particular, implies same expectation for equals. Then the outcome in the first round must be individually rational with respect to the continuation payoffs.

We are now ready to state our main result.

THEOREM 1. *Let $E = (N, u, a; c(\cdot))$ be an (excludable) public good economy satisfying assumptions* 1 *and* 2. *Then irrespective of any order w chosen in period* 1, *the unique (subgame perfect) equilibrium outcome of the demand-subscription game, Γ_2, leads to no exclusion. The implemented level of public good is the efficient level y^* and the resulting allocation of subscriptions is symmetric and stable (in the sense of the core).*

The following example illustrates why we need *sequential* mechanisms to establish the unique, efficient outcome(s) in Proposition 1 and Theorem 1. In the first two examples we discuss, whenever the agents move in sequence their names will also denote the order of moves, e.g., agent 1 moves first, agent 2 second etc.

Two-Agent Example. Consider two agents, each with utility function $u_i(y) = y^{1/2}$ and each having an access to a linear technology $c(y) = y$. The efficient level of production is $y^* = 1$. The question is, whether this efficient level can be induced in a unique equilibrium. It is easy to check that if agents are asked to announce their demand-subscription pairs simultaneously once-for-all (as opposed to sequential announcements in the one-shot game Γ_1) and the planner implements the maximal compatible coalition, then agent 1 bidding $(y_1, x_1) = (0, 0)$ and agent 2 bidding $(y_2, x_2) = (1/4, 1/4)$ is a Nash equilibrium that is also compatible and will be implemented by the planner. Clearly the outcome, $y = 1/4$, is inefficient. If, however, the agents announce sequentially according to Γ_1, bidding

$(y_1, x_1) = (0, 0)$ by agent 1 and bidding $(y_2, x_2) = (1/4, 1/4)$ by agent 2, though a Nash equilibrium, is not an SPE.[14] As we argue below, the only SPE will have agent 1 bid $(y_1, x_1) = (1, 1/4)$ and agent 2 bid $(y_2, x_2) = (1, 3/4)$.

Because agent 2's stand alone utility is 1/4, he will comply with any bid by agent 1 as long as it yields agent 2 a (net) utility of at least 1/4. Given this, agent 1 would target the production level y to maximize the overall social surplus $u_1(y) + u_2(y) - c(y)$, i.e., announce the efficient level of output $y_1 = 1$, and then choose as his share of the overall cost $c(1) = 1$ the minimal amount necessary to leave agent 2 a (net) utility exactly equal to 1/4. This implies $x_2 = 3/4$, $x_1 = 1/4$, so $(y_1, x_1) = (1, 1/4)$, $(y_2, x_2) = (1, 3/4)$ is an SPE. Finally, since no other combination of bids by agents 1 and 2 can yield agent 1 a (net) utility of 3/4 or more while guaranteeing agent 2 a (net) utility of 1/4, the above SPE is also unique.[15] ∎

The superiority of sequential-move demand-subscription games in this paper contrasts with Varian's [22] findings, in pure public good case, that a sequential-move direct contribution game will involve greater free-rider problem compared to a simultaneous-move direct contribution game. The intuition for this difference goes as follows: *demand-subscriptions* in our sequential mechanisms provide the agents moving early with an opportunity to make conditional commitments so that agents moving later cannot free ride which is not possible in simultaneous versions of the mechanisms; in Varian [22], sequential *contributions*, once sunk, deprive the early movers of the conditional commitment power, whereas for simultaneous contributions agents have *one* less opportunity to free ride because contributions are no longer sunk. The distinction between "subscription" and "contribution" in the context of free-riding in public goods has been previously emphasized by Admati and Perry [1].

To prove Theorem 1, we need to analyze the equilibrium behavior of the agents for the mechanism Γ_2. But then we need to analyze first the mechanism Γ_1 which is a proper subgame of Γ_2. So our approach in the rest of this section would be as follows: develop the necessary background for an analysis of our mechanisms, provide the formal proofs of

[14] Why? Agent 2's strategy of bidding $(y_2, x_2) = (1/4, 1/4)$ is not a Nash best response along all subgames including the ones not reached. In particular, along the subgame induced by agent 1 bidding $(y'_1, x'_1) = (1, 1/4 + \varepsilon)$, agent 2 would do better to bid $(y'_2, x'_2) = (1, 3/4 - \varepsilon)$ where $\varepsilon > 0$.

[15] If the game Γ_2 is played, the only SPE will have agent 1 bid $(y_1, x_1) = (1, 1/2)$ and agent 2 bid $(y_2, x_2) = (1, 1/2)$ in period 1 resulting in an overall agreement that is also efficient and the game does not proceed to period 2 (see the proof of Theorem 1). However, it is not difficult to see that if the agents announce simultaneously (as opposed to announcing sequentially) in the two periods of a game that is otherwise identical with the game Γ_2, there will be inefficiency.

Propositions 2, 1 and Theorem 1 in this specific order, and finally establish in Proposition 3 a desirable property of the mechanisms.

The analysis of demand-subscription games Γ_1 and Γ_2 will make use of the notion of the "greedy algorithm," previously used by Moulin [17] and Winter [23] in the context of Natural monopoly and by Moulin [14] in the context of public goods.

The Greedy Algorithm. Fix an arbitrary ordering of agents, denoted 1, ..., n. Then

- u_n^* is agent n's stand alone utility: $u_n^* = \max_{y \geqslant 0} u_n(y, c(y))$;
- u_{n-1}^* is agent $n-1$'s best utility such that (u_{n-1}^*, u_n^*) is feasible for coalition $\{n-1, n\}$ standing alone;
-
-
-
- u_1^* is agent 1's best utility such that $(u_1^*, ..., u_n^*)$ is feasible for the grand coalition.

Thus for a predetermined order of agents, greedy algorithm gives agent i a net utility equal to his marginal contribution in the coalition $\{i, i-1, ..., n\}$ as it expands from the smaller coalition $\{i-1, ..., n\}$; an agent fares better coming early than late in the order w.

Definition 4. The stand alone utility of a coalition $S \subset \{1, 2, ..., n\}$ is given by

$$\mathrm{sa}(S) = \max_{\langle y', x_j' \rangle} \sum_S u_j(y', x_j') \qquad \text{such that} \quad \sum_S x_j' = c(y'), \quad 0 \leqslant x_j' \leqslant a_j, \quad j \in S.$$

For the analysis to follow define

$$\begin{aligned} d_n &= \mathrm{sa}(\{n\}) \quad (= u_n^*), \\ d_k &= \mathrm{sa}(\{k, k+1, ..., n\}) - \mathrm{sa}(\{k+1, ..., n\}), \qquad k = 1, 2, ..., (n-1). \end{aligned} \tag{2}$$

In fact, since the public good game is a transferable utility game by assumption 2(b),

$$d_i = u_i^* \qquad \text{for all } i.$$

Also,

$$d_i \geqslant \mathrm{sa}(\{i\}) \qquad \text{for all } i.$$

The following result is due to Moulin [14, Theorem 5.2(a), pp. 281–283].

LEMMA 0. *The greedy algorithm utility vector* $\langle d_i \rangle$ *belongs to the core.*

Later on in the proof of Proposition 1 we will show that the mechanism Γ_1 induces the allocation defined by the greedy algorithm as its unique subgame perfect equilibrium. But before that we need a number of lemmas.

Since $y^* > 0$ and any core outcome is necessarily efficient, it must be that for at least one agent i in the order w, $d_i > 0$.

LEMMA 1. *Let* $d_i > 0$ *for some* i *in the order* w. *Then for all* $j < i$, $d_j > 0$ *and moreover* $d_j > \mathrm{sa}(\{j\})$.

Fix an order w. Let $h(w)$ be the highest $i < n$ in the order w with $d_i > 0$. Define $\mathscr{P} = \{h(w)+1, ..., n\}$, and $\mathscr{P}^C = N \setminus \mathscr{P}$.[16] Consider any set $S \subset N := \{1, 2, ..., n\}$, where $S \cap \mathscr{P}^C \neq \emptyset$.

LEMMA 2. *If* $\mathscr{P} \not\subset S$, *then* $\mathrm{sa}(S) < \sum_{i \in S} d_i$.

We note the following corollary to Lemma 2 for future reference.

COROLLARY 1. *Fix an order* w. *Then for agent* $h(w)$, $\mathrm{sa}(\{h(w)\}) < d_{h(w)}$.

Lemma 2 highlights the pivotal role of each of the "last few agents" (i.e., the members of $\mathscr{P}$) in inducing the grand coalition in the game Γ_1: Even if an agent j in $\mathscr{P}$ may not necessarily have a strictly positive marginal contribution in the greedy algorithm for the order w,[17] will, however, have an important contribution towards the overall utility of any group S (where $S \cap \mathscr{P}^C \neq \emptyset$) in the game Γ_1—if any agent j in $\mathscr{P}$ is missing from a coalition S in the game Γ_1, the coalition S will fail to achieve its greedy algorithm utility level $\sum_{i \in S} d_i$. Lemma 1 shows that any agent i (other than agent n) who has a positive utility in the greedy algorithm ($d_i > 0$) will get strictly less by standing alone in the game Γ_1. Thus the combined message of Lemmas 1 and 2 is simple: The agents heavily rely on each other, particularly on the last few agents,[18] for the realization of their mutual benefits; this acts as a bonding device making the task of coalition formation in the game Γ_1 relatively less difficult. Similar incentives for cooperation have been previously noted in different contexts by Winter [24] and Moldovanu and Winter [12].

Next we discuss a three-agent example to illustrate the workings of the sequential mechanisms in the coalition formation process. The

[16] Agent n always belongs to $\mathscr{P}$, while some others may or may not belong to $\mathscr{P}$.

[17] If $d_n > 0$ then $d_i > 0$ for all $i < n$ (by Lemma 1) in which case agent n is the only agent among the last few agents.

[18] The pivotal role of the last few agents will be crucial in the coalition formation process, as the proof of Proposition 1 will show.

insights gained from this example are further formalized and presented in Proposition 2.

Three-Agent Example. Instead of two agents in the previous example, we now have three agents, each with utility function $u_i(y) = y^{1/2}$ and each having access to the technology $c(y) = y$, play the game Γ_1. Demand-subscription announcement (y_1, x_1) by agent 1 induces a subgame along which agent 2 responds by announcing (y_2, x_2), and this is followed in the final subgame by agent 3 announcing (y_3, x_3). Start from the final subgame: agent 3, because of his stand alone (net) utility 1/4, will submit a bid compatible with previous bids as long as he gets 1/4 (net) utility out of it.

Agent 2's reasoning now goes as follows. Given an announced bid of (y_1, x_1) by agent 1, agent 2 will target the production level y that maximizes $u_2(y) + u_3(y) - c(y)$ and then decide on a subscription that will leave agent 3 a (net) utility 1/4. This means that if $y_1 \leqslant 1$ then agent 2 will propose $(1, x_2)$ such that $x_2 = 1 - x_1 - 3/4$. However, if $y_1 > 1$ then agent 2 reasons as follows: If there exist x_2 and x_3 such that $x_1 + x_2 + x_3 \geqslant y_1$ and $y_1^{1/2} - x_2 \geqslant 3/4$, $y_1^{1/2} - x_3 \geqslant 1/4$, then agent 2 bids the smallest x_2, satisfying these (inequality) restrictions, as his subscription towards y_1 (recall, 3/4 is the maximal net utility agent 2 can attain by cooperating with agent 3 alone); if, on the other hand, there exist no such x_2, x_3 then agent 2 bids (1, 1/4) and relies on agent 3's subscription of 3/4, in which case agent 1 will be excluded.

Finally, consider the decision process of agent 1 at the beginning of the game. Agent 1 will target the production level maximizing the total (net) utility of all three agents, $3y^{1/2} - y$, which gives $y_1 = y^* = 9/4$. Now to determine his own cost-share, agent 1 will leave a (net) utility of 3/4 to agent 2 and 1/4 to agent 3. So agent 1 can rely on a subscription of 3/4 by agent 2 and 5/4 by agent 3, and his remaining cost-share is only 1/4 and (net) utility 5/4, i.e., the equilibrium bids for the overall game Γ_1 are $(y_3, x_3) = (9/4, 1/4)$, $(y_2, x_2) = (9/4, 3/4)$, $(y_1, x_2) = (9/4, 5/4)$.

Thus, the coalition formation process unfolds from the final subgame backwards, the grand coalition forms and the final (net) utility vector is (5/4, 3/4, 1/4). ∎

Proposition 2 is an intermediate step to Proposition 1.

PROPOSITION 2. *Let $E = (N, u, a; c(\cdot))$ be an (excludable) public good economy satisfying assumptions* 1 *and* 2. *Fix an order w on N and the agents are re-named according to their positions in the order w. Now let $1 < k \leqslant h(w)$ and consider the subgame Γ_1^k (of the demand-subscription game Γ_1) starting with k's demand-subscription announcement after* $1, 2, ..., k-1$ *already announced their demand-subscription pairs* (y_1, x_1), (y_2, x_2), ...,

(y_{k-1}, x_{k-1}). *Suppose* (y_i, x_i) *are such that* $u_i(y_i) - x_i \geqslant d_i$ *for* $1 \leqslant i \leqslant k-1$. *Then all equilibria in the subgame* Γ_1^k *will have demand-subscription announcements* $p_j = (y_j, x_j)$ *such that* $u_j(y_j) - x_j = d_j$ *for* $j = k, ..., n$; *also the* p_j*'s are compatible, and the maximal compatible coalition in equilibrium along the subgame* Γ_1^k *must include the agents* $\{n, n-1, ..., k\}$.

Proof. We will use an induction argument. The original game Γ_1 is an extensive form game with a finite number of subgames, and the equilibrium result to be established is for the subgame Γ_1^k. There are two possibilities to consider: $h(w) = n-1$ and $h(w) < n-1$. Initially assume that $h(w) = n-1$ and use induction argument on k. First consider $k = n-1$.

Suppose $p_{n-1} = (y_{n-1}, x_{n-1})$ is such that $u_{n-1}(y_{n-1}) - x_{n-1} > d_{n-1}$. Since $\langle d_i \rangle$ belongs to core (Lemma 0), for p_{n-1} to be compatible with some other demand-subscription pair(s) it must be that $p_n = (y_n, x_n)$ satisfies $u_n(y_n) - x_n < d_n$. But this is not possible as agent n can guarantee himself d_n by standing alone.

Suppose $p_{n-1} = (y_{n-1}, x_{n-1})$ is such that $u_{n-1}(y_{n-1}) - x_{n-1} = d_{n-1}$. For such a claim by agent $n-1$ to be fulfilled, both agent n and agent $n-1$ must necessarily be included in the maximal compatible coalition to be chosen by the planner—this follows directly from Lemma 2.[19,20] The relevant question is whether agent $n-1$ can indeed expect to be included, and thus his claim fulfilled, in the maximal compatible coalition to be finally chosen by the planner. In the following we answer this to be in the positive.

Claim 1. Along all equilibria in the subgame starting with agent $n-1$'s move, agent $n-1$ will announce a suitable $p_{n-1} = (y_{n-1}, x_{n-1})$ with the net payoff d_{n-1}, and agent n will announce a suitable $p_n = (y_n, x_n)$ satisfying $u_n(y_n) - x_n = d_n$ so that both p_{n-1} and p_n are part of the maximal compatible coalition to be chosen by the planner.

Proof of Claim 1. To establish Claim 1, a number of subtle aspects need to be examined. Suppose agent $n-1$ announces a $p_{n-1} = (y_{n-1}, x_{n-1})$ such that $u_{n-1}(y_{n-1}) - x_{n-1} = d_{n-1}$. Since $\langle d_i \rangle$ belongs to core, given the announcements of the rest of the agents agent n can receive at most d_n; so, agent n may not (credibly) claim more than d_n.[21] Thus the possible (credible) strategies of agent n are:

[19] It is possible that this maximal compatible coalition consists of only agent n and agent $n-1$, if, say, agent $n-2$ claims strictly more than d_{n-2}; it is also possible that the maximal compatible coalition consists of some of the other agents as well.

[20] If not included in the maximal compatible coalition, agent $n-1$ receives his stand alone utility $\mathrm{sa}(\{n-1\})$ which is less than d_{n-1} (see Corollary 1).

[21] So claiming more than d_n cannot be used as a strategic threat by agent n; however, agent n may very well use the claim of d_n as a credible threat strategy, as it will be clear from the arguments to follow.

(a) Agent n chooses to stand alone[22] and receive only d_n, in which case the planner will not be able to recommend a compatible coalition and all the agents will receive only their respective stand alone utilities—a direct implication of Lemma 2;

(b) Agent n announces a demand-subscription pair so that the induced maximal compatible coalition will *exclude* agent $n-1$, but at the same time will include some (or all) agent(s) preceding agent $n-1$;

(c) Agent n announces a demand-subscription pair so that the induced maximal compatible coalition will *include* agent $n-1$ and agent n himself, and agent n receives only d_n.

The question is why should agent n choose (c), i.e., announce a demand-subscription pair inducing a maximal compatible coalition that includes agent $n-1$ with agent $n-1$ getting a payoff of d_{n-1} and agent n himself getting only d_n. Why not agent n instead choose to stand alone and get a payoff d_n as in (a), or alternatively induce a maximal compatible coalition that will exclude agent $n-1$ as in (b)? By choosing (b) (when such an alternative is available to agent n), agent n can never obtain strictly more than d_n (because $\langle d_i \rangle$ belongs to core).[23] On the other hand by choosing (a) (or sometimes (b), when alternative (b) gives agent n a (net) utility d_n), agent n can threaten agent $n-1$ to push him to his stand alone utility $\text{sa}(\{n-1\})$ which is less than d_{n-1} (by Corollary 1) and thus induce agent $n-1$ to concede agent n strictly more than d_n along the subgame starting with agent $n-1$'s move. Faced with this possibility agent $n-1$ would always prefer to concede agent n slightly more than d_n, say $\varepsilon > 0$, to get agent n to announce a demand-subscription pair that would induce a maximal compatible coalition including agent $n-1$ (Why? Agent $n-1$ himself will not be able to induce a maximal compatible coalition that would exclude agent n and receive a payoff close to d_{n-1}, as Lemma 2 shows; but he can achieve arbitrarily close to d_{n-1} by conceding agent n only small $\varepsilon > 0$.). But then agent $n-1$ will have no best response as he would like to concede agent n as little as possible. Thus strategy (a) or strategy (b) cannot be part of a subgame perfect equilibrium; agent n will choose strategy (c). So, along the subgame starting with agent $n-1$'s move, the maximal compatible coalition induced in equilibrium must contain $\{n, n-1\}$. (It is possible that announcement by agent n is such that the induced maximal compatible coalition will include some or all of the other agents preceding

[22] Though the stand alone alternative is not an explicit option for an agent when announcing his demand-subscription pair in the mechanism Γ_1 (or in the mechanism Γ_2), the agent can always announce an infeasible demand-subscription pair which is equivalent to standing alone.

[23] It is possible that agent n in fact gets less than d_n in which case strategy (b) is dominated by strategy (a).

agent $n-1$, as well.) The only other point to be clarified is how y_{n-1} and y_n compare. Since bidding is for public goods, agents $\{n, n-1\}$ will necessarily agree on a common level of the public good (existence guaranteed by the definitions $d_n = \text{sa}(\{n\})$, $d_{n-1} = \text{sa}(\{n-1, n\}) - \text{sa}(\{n\})$; see also in this context Moulin's [14, p. 282] construction of the subscription vector $(x_1^*, ..., x_n^*)$) so that $y_{n-1} = y_n$. This completes the proof of Claim 1.

Now assume that the Proposition is true for all j, $k < j < n$. Consider k's decision. If k announces $p_k = (y_k, x_k)$ such that $u_k(y_k) - x_k = d_k$, then by the induction assumption $p_j = (y_j, x_j)$ is such that $u_j(y_j) - x_j = d_j$ for all $j > k$, and the coalitions $\{n\}, \{n, n-1\}, ..., \{n, n-1, ..., k\}$ are all compatible since the agents in a coalition will choose suitably the same public good level. Now apply the lack-of-best-response argument as follows. Coalition-payoff combination $\{n; d_n\}$ cannot be part of a subgame perfect equilibrium (see the argument in the previous paragraph used to establish Claim 1). Similarly, coalition-payoff combination $\{n, n-1; d_n, d_{n-1}\}$ cannot be part of a subgame perfect equilibrium. Why? Agent $n-2$ would rather accept $d_{n-2} - \varepsilon$ and concede $\varepsilon > 0$ to the last two agents to induce them to announce demand-subscription pairs leading to a maximal compatible coalition that includes agent $n-2$;[24] by Lemma 2, agent $n-2$ by standing alone will not be able to get a payoff close enough to d_{n-2}. But then agent $n-2$ prefers conceding as little ε as possible. Apply similar reasoning to conclude that the maximal compatible coalition in equilibrium along the subgame Γ_1^k must include the agents $\{n, n-1, ..., k\}$. (It is possible that the planner will be able to include, in the maximal compatible coalition, some or all of the other agents preceding agent k, as well.) Also, these agents will necessarily agree on a common level of public good so that $y_k = \cdots = y_{n-1} = y_n$. So announcing $p_k = (y_k, x_k)$ such that $u_k(y_k) - x_k < d_k$ is ruled out. Ruling out an announcement $p_k = (y_k, x_k)$ such that $u_k(y_k) - x_k > d_k$ is easy: by the induction assumption, $p_j = (y_j, x_j)$ is such that $u_j(y_j) - x_j = d_j$ for all $j > k$; since $\langle d_i \rangle$ belongs to core, p_k will not be compatible with any other demand-subscription announcement(s) nor can it be achieved by agent k standing alone.

Finally, relax the assumption that $h(w) = n-1$. Suppose $h(w) < n-1$. Now go back to our argument for the case when $h(w) = n-1$. There the assumption $h(w) = n-1$ has been used in two instances: first (refer footnote 20), to ensure that agent $n-1$ will not be able to realize $d_{n-1} > 0$ by standing alone because $\text{sa}(\{n-1\}) < d_{n-1}$, so agent $n-1$ must announce suitably such that both agent $n-1$ and agent n are included in the maximal compatible coalition chosen by the planner; second (refer the argument establishing Claim 1), to argue that agent $n-1$ (resp. similar

[24] Without this ε-incentive, one of the last two agents can always announce demand-subscription pairs and induce a maximal compatible coalition that leave out agent $n-2$.

other agents $j < n-1$ in the backwards-induction chain) would rather concede agent n (resp. the group of later agents $j+1, ..., n$ to follow) some ε. Now in both the instances described above what is important is that agent $n-1$ (resp. similar other agents in the backwards-induction chain) be confronted with some strictly positive gain, like d_{n-1}, that he cares about, but which cannot be realized unless the maximal compatible coalition induced includes also agent n (resp. all the agents to follow including agent n). Now when $h(w) < n-1$, $d_j = 0$ for $j = h(w)+1, ..., n$. But then, as Lemma 2 shows, agent $h(w)$'s strictly positive marginal gain, $d_{h(w)}$ ($> \text{sa}(\{h(w)\})$), cannot be arbitrarily closely realized unless agent $h(w)$ can also induce a maximal compatible coalition that will include each single agent following him all the way to agent n (i.e., the agents $h(w)+1, h(w)+2, .., n$). This can be done if agent $h(w)$ concedes in the form of an incentive a strictly positive ε_1 to the agents to follow: once $\varepsilon_1 > 0$ trickles down to agent $h(w)+1$, he will pass a fraction of ε_1, say $\varepsilon_2 = \gamma\varepsilon_1 > 0$, to agent $h(w)+2$, etc. So the arguments used to establish the Proposition for $h(w) = n-1$, continue to hold for the case when $h(w) < n-1$. Q.E.D.

We now provide the formal proofs of Proposition 1 and Theorem 1.

Proof of Proposition 1. Consider agent 1's announcement decision. If agent 1 announces $p_1 = (y^*, x_1)$ such that $u_1(y^*) - x_1 = d_1$, then by Proposition 2 agent j will announce $p_j = (y^*, x_j)$ satisfying $u_j(y^*) - x_j = d_j$, $2 \leqslant j \leqslant n$ and the (only) maximal compatible coalition induced in equilibrium is the grand coalition (by the lack-of-best-response argument). So announcing $p_1 = (y_1, x_1)$ such that $u_1(y_1) - x_1 < d_1$ is not optimal for agent 1. Announcing $p_1 = (y_1, x_1)$ such that $u_1(y_1) - x_1 > d_1$ is not optimal for agent 1 either, because then by Proposition 2 the maximal compatible coalition induced in equilibrium along the subgame Γ_1^2 and implemented by the planner will exclude agent 1.

In particular, announcement of any $y \neq y^*$ by agent 1 will fail to yield d_1 for agent 1. Why? Such an y is inefficient and not in the core. So for agent 1 to receive d_1, some other agent must accept less than his greedy algorithm utility. Let j be the first such agent to receive less than d_j. But by Proposition 2, agent j can always guarantee d_j by announcing a suitable y' that achieves $\text{sa}(\{j, j+1, ..., n\})$ and inducing the agents following j to coordinate on y'—thus contradicting that agent 1 can obtain d_1 by announcing $y \neq y^*$. So agent 1 will announce the efficient level, y^*. Similarly agents $2, ..., n$ will announce y^*—incentives for inefficient y-announcements by the "last few agents" are ruled out in subgame perfect equilibrium by an argument very similar to the one in Proposition 2 ruling out the stand alone alternative for agent n.

So the sequential announcements of $p_j = (y^*, x_j)$ satisfying $u_j(y^*) - x_j = d_j$, $1 \leqslant j \leqslant n$ and the implementation by the planner of the grand coalition as the maximal compatible coalition is a subgame perfect equilibrium of the game Γ_1. It can be verified that $x_j = x_j^*$, $1 \leqslant j \leqslant n$, where x_j^*'s are the greedy algorithm subscriptions corresponding to y^*—see the construction of the x_j^*'s in Moulin's [14, pp. 281–283] proof of Theorem 5.2. The uniqueness of the equilibrium follows from the uniqueness of the efficient level y^* and the assumption that $u_j(y, x_j)$ is increasing in y and decreasing in x_j for all j (Assumption 2(b)). To summarize: The game Γ_1 uniquely implements the efficient level y^*, results in an allocation of subscriptions that gives rise to the core utility vector $\langle d_i \rangle$, and no agent is excluded in equilibrium. Q.E.D.

Proof of Theorem 1. Suppose the game Γ_2 proceeded to period 2 and consider this second-period-play subgame from an agent's point of view.

For each order w and agent j in N, let $w(j)$ be j's place in the order w. Define $\Omega_{j,i} = \{w : w(j) = i\}$, i.e., the set of all orders with agent j in the ith place (the cardinality of $\Omega_{j,i}$ equals $(n-1)!$, as there are $(n-1)!$ ways of having agent j in the ith place). Now, for a specific realized order w in period 2, calculate agent j's payoff who is in place $w(j) = i$. Since period 2 is the final period, payoffs are calculated exactly as in Proposition 1: Agent j receives exactly equal to his marginal contribution $d_{j,i}$ in the order w (see the definitions of d_i's in (2)).[25] In fact, $d_{j,i}$ should further have the order w as an argument because the marginal contribution of an agent depends on the specific order: $d_{j,i}(w)$. Now step back to just when an order w is to be drawn and calculate j's (ex-ante) expected payoff in period 2 play as follows:

$$\lambda^j = \frac{[D_{j,1} + D_{j,2} + \cdots + D_{j,n}]}{n}, \qquad \text{where} \quad D_{j,i} = \frac{1}{(n-1)!} \sum_{w \in \Omega_{j,i}} d_{j,i}(w).$$

($D_{j,i}$ is the expected payoff of agent j conditional on appearing ith in the order.) Agent j is just a metaphor for any agent, and because the (net) utility vector $\langle d_i \rangle$ belongs to core and the core is convex (due to transferable utility assumption), the (ex-ante) expected payoff vector for period 2 play, denoted by $\langle \lambda^j \rangle$, corresponds to a *unique* (interior) name-specific core outcome $\langle y^*; (\hat{x}_j)_{j \in N} \rangle$ of public good level and individual subscriptions irrespective of the order w.[26] Moreover, since all agents get an equal chance of being selected in each specific slot/rank in the random draw for the second-period play of the game Γ_2, the agents with identical preferences will receive exactly the same (net) expected utility in the

[25] To be precise, (2) defines only d_i which is the greedy algorithm utility of an agent in place i for a given order w; $d_{j,i}$ is same as d_i, with the additional subscript j now explicitly denoting the agent's real name.

[26] The *uniqueness* follows from the uniqueness of the efficient level y^* and the assumption that $u_j(y, x_j)$ is increasing in y and decreasing in x_j for all j.

second-period-play subgame. This in turn implies that the agents with identical preferences will have exactly the same payoff in the core outcome $\langle y^*; (\hat{x}_j)_{j\in N}\rangle$, which further implies that they will make the same individual subscriptions. Thus the specific core outcome, $\langle y^*; (\hat{x}_j)_{j\in N}\rangle$, is symmetric (refer Definition 3).

Now go back to period 1 play with *any* predetermined order w'. An agent (or player) in the order w', while deciding his optimal strategy in period 1, must account for the likely outcomes for each of the players in the game (including himself) if the game were to proceed to period 2. The first-period play (of the overall game Γ_2) is now very much like the one-shot play of the game Γ_1 as in Proposition 1, except that now the players' stand alone payoffs are determined by the vector $\langle \lambda^j \rangle$. [Why? If any player j receives less than λ^j, he will veto the outcome in period 1 play and the game would proceed to period 2 that will give him an expected utility of λ^j.] Now apply Proposition 2 and Proposition 1 with $\langle \lambda^j \rangle$ replacing the corresponding elements of $\langle d_i \rangle$: Agent 1 (whose real name is Leo, say) in the order w will announce $p_1=(y^*, \hat{x}_1)$ satisfying $u_1(y^*)-\hat{x}_1=\lambda^{\mathrm{L}}$; agent 2 (whose real name is Brian, say) will announce $p_2=(y^*, \hat{x}_2)$ satisfying $u_2(y^*)-\hat{x}_2=\lambda^{\mathrm{B}}$; etc. Thus the net utility vector $\langle \lambda^j \rangle$ is realized in period 1 itself which is a subgame perfect equilibrium of the game Γ_2, and our claims are almost established. Why almost? Still we need to explain why an agent may simply be content getting ex-post in period 1 a payoff equal to the (ex-ante) expected payoff from period 2 play, and not prefer to proceed to period 2. Here we assume that all agents (even when they are risk-neutral, as we have assumed) have a lexicographic preference for period 1 settlement over the option of going to period 2 and realizing the same utility ex-ante. (Alternatively one may assume that the agents discount the future mildly.) Thus the subgame perfect equilibrium $\langle y^*; \hat{x}_1, ..., \hat{x}_n\rangle$,[27] with the implementation by the planner of the grand coalition in period 1 as the maximal compatible coalition, is unique: since the game does not proceed to period 2, invoke the same uniqueness argument as in Proposition 1. Implementation of the grand coalition implies no-exclusion. Already we have shown that the ex-post outcome $\langle y^*; \hat{x}_1, ..., \hat{x}_n\rangle$ is symmetric. Also, the equilibrium output y^* is efficient. Q.E.D.

The following corollary is a direct implication of Theorem 1.

COROLLARY 2. *The symmetric outcome implemented in Theorem* 1 *is same as the Shapley value solution* (*Shapley* [21]) *of the underlying characteristic function game defined by* (2).

[27] Agents with index 1,2...,n for a predetermined order w will have their corresponding real names Leo, Brian etc.

In the Appendix we demonstrate, in a non-symmetric and extended version of our previous two-agent example, how the Shapley value arises from our mechanism.

Remark. First, Shapley value solution is considered to be a standard equity criterion in the fair-division literature [14, p. 403], hence the mechanism Γ_2 achieves something more than horizontal equity. Second, although in the mechanism Γ_2 we use a random device in determining the order of play in the second-stage to eliminate the early-mover-advantage in the first-stage play, the random order selection rule is never used ex-post because the grand coalition forms in stage 1 itself.[28] Third, the importance of excludability to implement the efficient public good level and thus overcome the free-rider problem is worth emphasizing. Minus this characteristic, our demand-subscription game is ineffective. Some of the other works in excludable public good setting (e.g., Moulin [15], Deb and Razzolini [7]) sacrifice efficiency to explore a different but equally interesting and more demanding aspect of the demand-revelation issue, that of strategy-proofness.

Another desirable property of the mechanisms Γ_1 and Γ_2 is the fact that the equilibria implementing the public good outcome are also "strong" in the sense of Aumann [3].

DEFINITION 5 (Aumann [3]). A *strong equilibrium* in an n-player game is an n-tuple of strategies for which no coalition of players can simultaneously all do better for themselves by moving to different strategies, while the players outside of the coalition maintain their original strategies.

PROPOSITION 3. *The subgame perfect equilibria of Γ_1 and Γ_2 and of all the subgames, Γ_1^k, starting with agent k's announcement in the game Γ_1, are strong equilibria.*

Proof. First consider the equilibria along the subgame Γ_1^k. For $k = n$, the only coalition possible is the single-agent coalition $\{n\}$. By Proposition 2 agent n cannot improve his payoff over d_n by switching from his equilibrium strategy to another strategy, hence the (strong-equilibrium) hypothesis holds for the final subgame Γ_1^n. Now assume that the hypothesis

[28] Instead of using the random device as in our mechanism Γ_2, the order of moves can possibly be endogenized as follows: conduct a sealed-bid conditional all-pay auction for each possible order simultaneously and independently, choose the order with the highest sum total of bids proviso the sum is nonnegative (individual bids can be negative or positive) and ask the players to pay their bids for this particular order (a player making a negative bid in fact receives a transfer), and play the game Γ_1 only once with respect to this order. However, whether such an auction mechanism or some variation of it can induce a desirable core outcome is an open question. In contrast to the simple mechanisms in this paper, auctioning the order-of-play would enhance strategic complexity.

is true for $k=j$, $j<n$, and consider $k=j-1$, i.e., the subgame Γ_1^{j-1}. Agents $i=1, ..., j-2$ already announced demand-subscription pairs (y_i, x_i) such that $u_i(y_i)-x_i \geqslant d_i$. If any coalition $\mathscr{C} \subset \{j-1, j, ..., n\}$ improves, contrary to Proposition 3, by switching to some non-equilibrium strategies, then $j-1$ must have chosen a strategy different from his equilibrium strategy and hence is part of the coalition $\mathscr{C}$. (If $j-1$ stayed with his equilibrium strategy which meant announcing (y_{j-1}, x_{j-1}) such that $u_{j-1}(y_{j-1})-x_{j-1}=d_{j-1}$, then the profitable coalitional deviation by $\mathscr{C}$ would contradict the assumption that the hypothesis is true for $k=j$.) In particular, $j-1$ must have announced some $(\hat{y}_{j-1}, \hat{x}_{j-1})$ such that $u_{j-1}(\hat{y}_{j-1})-\hat{x}_{j-1}>d_{j-1}$. Also $\mathscr{C}$ must contain at least one member from the set $\{j, ..., n\}$. Because if all members in $\{j, ..., n\}$ stayed with their original equilibrium strategies specified in Proposition 2 and $j-1$ received more than d_{j-1} by announcing $(\hat{y}_{j-1}, \hat{x}_{j-1})$, then the equilibrium strategy of $j-1$ in Proposition 2 is contradicted. Let $l \in \{j, ..., n\}$ be one such agent who is a member of the deviating coalition $\mathscr{C}$ and so receives more than d_l. But this then would contradict that the (subgame perfect) equilibria of the subgame Γ_1^j are strong equilibria. So it must be that the hypothesis is true for $k=j-1$ as well. Already we have shown that the hypothesis is true for $k=n$. So, by induction, our proposition is established for the subgame perfect equilibria of all the subgames Γ_1^k.

By a similar argument, the respective (unique) subgame perfect equilibrium outcomes of the games Γ_1 and Γ_2 are also strong equilibrium. Q.E.D.

4. CONCLUSION

In this paper we have proposed two sequential mechanisms that induce two very different core outcomes—one extreme outcome, another non-extreme (i.e., interior) outcome. We do not suggest that a planner should use the first mechanism for actual policy purposes as it involves discrimination between agents ex-post; this mechanism is only an intermediate step to a second mechanism that is more equitable. The second mechanism is very much like a twofold repetition of the first mechanism, is simple enough to apply, is finite, and treats agents symmetrically ex-post. Finally, although the public good is excludable and the mechanisms in this paper recommend exclusion under some circumstances, in equilibrium exclusion does not occur and only the grand coalition forms.

APPENDIX

Proof of Lemma 1. Let $\hat{y}>0$ achieve $\mathrm{sa}(\{i, i+1, ..., n\})$. Consider $j=i-1$. Since $u_{i-1}(y, x_{i-1})$ is increasing in y and $u_{i-1}(0)=0$, hence

$u_{i-1}(\hat{y}, 0) > 0$. Let $y_{i-1} \geqslant 0$ be agent $i-1$'s stand alone choice of the public good level and $\text{sa}(\{i-1\}) \geqslant 0$. Now expand the coalition $\{i, i+1, ..., n\}$ by including agent $i-1$. The expanded coalition can always produce and consume $\tilde{y} = \max\{\hat{y}, y_{i-1}\}$ with a cost $c(\tilde{y}) = \max\{c(\hat{y}), c(y_{i-1})\} \leqslant c(\hat{y}) + c(y_{i-1})$ so that $d_{i-1} = \text{sa}(\{i-1, i, i+1, ..., n\}) - \text{sa}(\{i, i+1, ..., n\}) > \text{sa}(\{i-1\}) \geqslant 0$. The same argument extends to all $j < i-1$. Q.E.D.

Proof of Lemma 2. Suppose $\mathscr{P} \not\subset S$. Since $\langle d_i \rangle$ belongs to core, $\text{sa}(S) > \sum_{i \in S} d_i$ is not possible. Alternatively assume that $\text{sa}(S) = \sum_{i \in S} d_i$. This, we will argue, is also impossible.

First consider $d_n > 0$. By Lemma 1 $d_j > 0$ for all j, hence $\mathscr{P} = \{n\}$. Thus agent n does not belong to the set S. By definition $\text{sa}(\{n\}) = d_n$. Let $y_n > 0$ be agent n's stand alone choice of the public good level and $\tilde{x}_n = c(y_n)$. Since $\sum_{i \in S} d_i > 0$, stand alone choice of the public good level by the coalition S must be positive: $y^S > 0$. Choose the (input) subscriptions $\tilde{x}_i$, $i \in S$, with $\sum_{i \in S} \tilde{x}_i$ producing y^S, such that each i receives exactly his greedy algorithm utility d_i. Now expand the coalition S to S' by including agent n, produce $y^+ = \max\{y^S, y_n\}$, and allocate the cost $c(y^+) = \max\{c(y^S), c(y_n)\} < c(y^S) + c(y_n)$ in such a way that the subscriptions of the members of S remain unchanged at their original levels, $x_i = \tilde{x}_i$, and agent n subscribes only $x_n = c(y^+) - c(y^S) < \tilde{x}_n$. Thus each member in the expanded coalition S' receives at least as much utility as his share in the core utility vector $\langle d_i \rangle$ and agent n receives more than d_n, contradicting that $\langle d_i \rangle$ belongs to core.

Now consider $d_n = 0$. Suppose some agent k in $\mathscr{P}$ does not belong to S. By definition, $\text{sa}(\{k\}) = d_k = 0$. $y^S > 0$ because $\sum_{i \in S} d_i > 0$ (recall, $S \cap \mathscr{P}^C \neq \varnothing$). So choose the subscriptions $\tilde{x}_i$, $i \in S$, with $\sum_{i \in S} \tilde{x}_i$ producing y^S, such that each i receives exactly his greedy algorithm utility d_i. Now expand the coalition S to $S^{\mathscr{E}}$ by merely including agent k and allowing him to consume y^S without having to subscribe anything. This coalition $S^{\mathscr{E}}$ thus blocks the core allocation $\langle d_i \rangle$, a contradiction.

So it must be that $\text{sa}(S) < \sum_{i \in S} d_i$. Q.E.D.

Example: Shapley Value Outcome. Consider a three-agent example: agents $i = 1, 2$ have utility function $u_i(y) = y^{1/2}$, agent 3's utility function is $u_3(y) = 2y^{1/2}$, and the technology is $c(y) = y$. In the following, first we calculate the agents' equilibrium bids for each order of play in the second stage of the game Γ_2.

Consider the order $w_1 = 3, 2, 1$ and analyze the moves in the reverse order. Agent 1 calculates his stand alone payoff by maximizing $u_1(y) - c(y) = y^{1/2} - y$, which gives $y_1^{\text{sa}} = 1/4$ and $\text{sa}(\{1\}) = 1/4$. So agent 1 would cooperate with any bid by agent 2 and agent 3 that gives him a payoff at least 1/4. Next agent 2 calculates his marginal contribution $\text{sa}(\{2, 1\}) - \text{sa}(\{1\})$ first by maximizing $u_1(y) + u_2(y) - c(y) = 2y^{1/2} - y$ and then leaving a (net) utility 1/4 to agent 1. This produces a payoff of 3/4 to agent 2, so now agent 2 would cooperate with agent 3's bid as long he gets at least 3/4. Finally agent 3 calculates his marginal contribution $\mathrm{sa}(\{3,2,1\}) - \mathrm{sa}(\{2,1\})$ by maximizing $u_1(y) + u_2(y) + u_3(y) - c(y) = 4y^{1/2} - y$ (thus selecting the unique, efficient output $y^* = 4$) and then leaving (net) utilities 1/4 to agent 1 and 3/4 to agent 2. This produces compatible bids $(y_3, x_3) = (4, 1)$, $(y_2, x_2) = (4, 5/4)$ and $(y_1, x_1) = (4, 7/4)$, resulting in net payoffs $(v_1, v_2, v_3) = (1/4, 3/4, 3)$. By symmetry between agents 1 and 2, equilibrium bids for the order $w_2 = 3, 1, 2$ are $(y_3, x_3) = (4, 1)$, $(y_1, x_1) = (4, 5/4)$ and $(y_2, x_2) = (4, 7/4)$, and the net payoffs are $(v_1, v_2, v_3) = (3/4, 1/4, 3)$.

By a similar exercise, equilibrium bids for the order $w_3 = 2, 3, 1$ are $(y_2, x_2) = (4, 1/4)$, $(y_3, x_3) = (4, 2)$ and $(y_1, x_1) = (4, 7/4)$ resulting in net payoffs $(v_1, v_2, v_3) = (1/4, 7/4, 2)$. By symmetry between agents 1 and 2, equilibrium bids for the order $w_4 = 1, 3, 2$ are $(y_1, x_1) = (4, 1/4)$, $(y_3, x_3) = (4, 2)$ and $(y_2, x_2) = (4, 7/4)$, and the net payoffs are $(v_1, v_2, v_3) = (7/4, 1/4, 2)$.

Similarly, equilibrium bids for the order $w_5 = 2, 1, 3$ are $(y_2, x_2) = (4, 1/4)$, $(y_1, x_1) = (4, 3/4)$ and $(y_3, x_3) = (4, 3)$ resulting in net payoffs $(v_1, v_2, v_3) = (7/4, 5/4, 1)$. By symmetry between agents 1 and 2, equilibrium bids for the order $w_6 = 1, 2, 3$ are $(y_1, x_1) = (4, 1/4)$, $(y_2, x_2) = (4, 3/4)$ and $(y_3, x_3) = (4, 3)$, and the net payoffs are $(v_1, v_2, v_3) = (5/4, 7/4, 1)$.

All orders are equally likely. Now calculate agent 1's expected payoff from stage 2 play of Γ_2: $(1/6)[v_1(w_1) + v_1(w_2) + v_1(w_3) + v_1(w_4) + v_1(w_5) + v_1(w_6)] = (1/6)[(1/4) + (3/4) + (1/4) + (7/4) + (7/4) + (5/4)] = 1$. By symmetry, agent 2's expected payoff is also 1. Agent 3's expected payoff is $(1/6)[v_3(w_1) + v_3(w_2) + v_3(w_3) + v_3(w_4) + v_3(w_5) + v_3(w_6)] = (1/6)[3 + 3 + 2 + 2 + 1 + 1] = 2$.

Now going back to stage 1 of Γ_1, for any order of play the equilibrium bids are always $(y_1^*, x_1^*) = (4, 1)$, $(y_2^*, x_2^*) = (4, 1)$, $(y_3^*, x_3^*) = (4, 2)$ so that the resulting payoff vector is $(v_1^*, v_2^*, v_3^*) = (1, 1, 2)$. Next we show that this payoff vector corresponds to the Shapley value.

The characteristic function of the three-agent cooperative game, defined by the technology and the preferences, is given by

$$\mathrm{sa}(\{1\}) = \mathrm{sa}(\{2\}) = 1/4, \qquad \mathrm{sa}(\{3\}) = 1,$$

$$\mathrm{sa}(\{1,2\}) = 1, \qquad \mathrm{sa}(\{1,3\}) = \mathrm{sa}(\{2,3\}) = 9/4,$$

$$\mathrm{sa}(\{1,2,3\}) = 4.$$

It is straightforward to check that the Shapley value is (1, 1, 2).

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