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"I DON'T WANT TO KNOW !" : CAN IT BE RATIONAL?

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"I don't want to know !": Can it be rational?

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Abstract

In this paper we will show that the usually accepted principle of decision theory that "the more information the better" seemingly breaks down in strategic contexts. We will show through several examples that almost every situation is conceivable: Information can be beneficial for all the players, or only for the one who receives it, or, less intuitively, just for the one who does not receive it, or it could be bad for both. The only class of games that escapes these seemingly surprising phenomena is the class of zero-sum games, but only under the assumption of common beliefs for the players. We will show that even a minor departure from the assumptions of zero-sum and common beliefs can produce the phenomenon of information-rejection. We will show that these phenomena may appear even in coordination games, where one would expect that public information should facilitate coordination. It should be emphasized that there is here neither a pathology nor a paradox: aside from the particular examples that may merit attention, the message is that in an interactive decision framework with incomplete information, the relevant issue is that of *interactive knowledge* rather than simply knowledge per se.

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1 Introduction

The statement "the more information the better" is commonly accepted both on the intuitive level and more formally in decision theory. The formalization of this idea goes back to Ramsey, whose note on the topic has been recently published (Ramsey (1990)). Other classical references are Blackwell (1951), (1953), Raiffa and Schlaifer (1961), Good (1967) Torgersen (1991a), (1991b), (1994). One formal expression of this principle is, for instance, as follows: Let $\{X_n\}$ be an infinite sequence of exchangeable random variables. By de Finetti's representation theorem (see de Finetti (1937), and, for a more recent reference, Schervish (1995)) there exists a random variable Θ such that the X_n are i.i.d. given Θ. The sequence $\Theta_n^* := E[\Theta|X_1, \ldots, X_n]$ is a Lévy martingale, therefore for every $n \in \mathbb{N}$, Θ_n^* is more disperse than Θ_{n+1}^* and any risk averse decision maker would rather make her decision based on Θ_{n+1}^* than on Θ_n^* and therefore will ask for more information (i.e. more observations), provided its cost is small enough. In particular she will always ask for more information, when this is free.

Exceptions emerge when one deviates from standard assumptions. Kadane, Schervish and Seidenfeld (1996) have shown that a Bayesian decision maker may rationally pay not to see the outcome of a certain cost-free experiment, when only finite additivity of the probability measures is assumed. Safra and Sulganik (1995) have dealt with similar phenomena for decision makers whose choice criterion is not the maximization of expected utility.

Even if we consider σ-additive probability measures and expected utility maximizers, the idea that the more information the better loses its appeal in interactive decision situations (games) involving more than one decision maker. Several seemingly counter-intuitive phenomena arise. For instance Bassan and Scarsini (1995), (1997), have considered a situation where different agents have to make a binary choice and each of them maximizes his expected utility according to his own subjective probability. Before the agents make their decision, information is revealed to all of them for free and without censoring, but the flow of information can be stopped by a planner whose aim is to maximize social expected utility. The social utility function is increasing and symmetric in the utilities of the agents (to represent benevolence and impartiality of the planner). Given the description of the situation, it would appear natural that, since information is beneficial for each agent, and since the well being of each agent is beneficial for the society, then information should be socially useful, and therefore the planner should never stop the flow of information. However this is not always the case. If the social utility function is concave in the sum of the utilities of the agents, then stopping the flow of information can sometimes be optimal for the planner. The key to this kind of phenomenon is that if $f(X, Y)$ is a function increasing in both variables then, when X and Y are random variables, the expectation of f is not necessarily increasing in the expectations of X and Y.

The role of information in strategic conflicts has been investigated in Kamien, Tauman and Zamir (1990), and its value has been studied by analyzing the effect of a hypothetical external agent, the "maven", endowed with some relevant information. The information held by the "maven" gives him the power of changing the structure of the game by transmitting information, possibly in a partial and private way. Thus, a family of games emerges, indexed by the signal released by the "maven". Special cases are those obtained when full information is transmitted to none, only one, or to a specific subset of players (in particular, to all of them).

In a two-person zero-sum game with common beliefs, it is not possible that both players strictly reject public information. The value of the new game resulting from any public disclosure of information is either (weakly) higher or lower than the original game; thus, it is not possible that both the maximizer and the minimizer strictly prefer to play the original game.

In this note we will show, by providing suitable examples, that except in two-person zero-sum games with common beliefs, it is always possible that both players prefer to be (both) uninformed rather than (both) informed. In particular, almost every situation is conceivable: Information can be beneficial for all the players, or only for the one who receives it, or, less intuitively, just for the one who does not receive it, or it could be bad for both.

We emphasize that there is here neither a pathology nor a paradox: aside from the particular examples that may be worth some attention, the message is: In an interactive decision framework with incomplete information, the relevant issue is that of *interactive knowledge* rather than simply the knowledge per se about the relevant parameters of the game. In other words, the effect of an additional information, public, private or any other form, is determined not only by the information as such but by its effect on the mutual knowledge situation: what each player knows about the information of each of the other players, what he or she knows about others' information on his information etc.

It should also be mentioned that Neyman (1991) showed that it is in fact impossible to compare two games G_1 and G_2 and say that in G_1 all players, except one, have the same information as in G_2. The comparison has to be made *within the same Harsanyi game*, in different states of the world. When this is done properly, it can be shown that in fact *information always has a positive value.* This result thus formalizes the statement that our examples exhibit no paradox, but it does not diminish their peculiarity. Furthermore, we believe that these examples capture real life phenomena.

Interesting considerations about the role of information in economic applications can be found for instance in Hirshleifer (1971), Hirshleifer and Riley (1979), Admati and Pfleiderer (1986).

In Section 2 non-zero-sum games will be considered, and in Section 3 zero-sum games with different beliefs, both consistent and inconsistent, will be dealt with. In Section 4 we show that even a minor perturbation of the zero-sum-game-with-common-beliefs scheme leads to the possibility of information rejection. In Section 5 a coordination game will be exhibited, in which both players prefer to be uninformed than informed. This is somehow counterintuitive, since one would except that public information should facilitate coordination.

2 Non-zero-sum games

In this section we provide a series of examples exhibiting various effects of information. In each of these examples a non-zero sum game will be considered. The basic solution concept in non-zero-sum games is the *Nash Equilibrium*. Since we consider two-person matrix games, Nash Equilibrium always exists (in mixed strategies). Typically, there are several equilibria and there is no "obvious outcome" of the game. To make our points regarding the role of information more convincing, we base our examples, mostly, on games in which each player has a *strongly dominant strategy* (i.e. a strategy which is always strictly better that all his other strategies no matter what the other player does). In such games (at least when they are played once), there is a convincingly clear outcome of the game: The result of each player choosing his dominant strategy. To emphasize this point we shall in fact use in this case the expression *"the outcome of the game"*.

In all the following examples, nature chooses one of the two matrices G_A, G_B with probability 1/2, the interpretation being that the state of nature is either A or B with equal probabilities. If the state is A (respectively: B), the payoff matrix is G_A (respectively: G_B). We shall refer to G_A and G_B as *state-games*. The state-games are given in normal (strategic) form where player 1 chooses the row and player 2 chooses the column (the choices are made simultaneously). An entry (a, b) represents a payoff of a units to the row player and b units to the column player.

Example 1.

$$G_A = \begin{pmatrix} 0,0 & 6,-3 \\ -3,6 & 5,5 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -20,-20 & -7,-16 \\ -16,-7 & -5,-5 \end{pmatrix}.$$

We refer to the rows as T (top) and B (bottom) and to the columns as L (left) and R (right). The two arrays are common knowledge, and so is the fact that nature chooses one of them with probability 1/2. First of all notice that in G_A the top row, T, strongly dominates the bottom row, B, and the left column, L, strongly dominates the right column, R. In G_B the situation is reversed: B strongly dominates T and R strongly dominates L. Therefore $(0,0)$ is the unique Nash equilibrium payoff in G_A and $(-5,-5)$ is the unique Nash equilibrium payoff in G_B. From this it follows that, if, before the players make their move, the state-game is revealed to both of them, they expect a payoff equal to

$$\frac{1}{2}(0,0) + \frac{1}{2}(-5,-5) = (-2.5,-2.5).$$

If it is common knowledge that both players are uninformed about the state-game that is being played, they act as if they were playing the game

$$\frac{1}{2}G_A + \frac{1}{2}G_B = \begin{pmatrix} -10,-10 & -0.5,-9.5 \\ -9.5,-0.5 & 0,0 \end{pmatrix}.$$

In this game B strongly dominates T and R strongly dominates L. Therefore the unique Nash equilibrium payoff is $(0,0)$.

If it is common knowledge that the state-game is revealed only to player I, then player II will expect player I to choose T in G_A and B in G_B. Therefore the payoffs are

$$(0,0 \quad 6,-3) \qquad \text{with probability } 1/2,$$

and

$$(-16,-7 \quad -5,-5) \qquad \text{with probability } 1/2;$$

hence she will have to choose left or right in the following row of expected payoffs

$$(-8,-3.5 \quad 0.5,-4).$$

The outcome of the game is now $(-8,-3.5)$.

By using the symmetry of the games, we can see that, if only player II is informed, and this is common knowledge, then the outcome is $(-3.5,-8)$.

If we summarize these results in what we shall refer to as the I-U (Informed-Uninformed) matrix,

	2-Inf	2-Uninf
1-Inf	$-2.5,-2.5$	$-8,-3.5$
1-Uninf	$-3.5,-8$	$0,0$

we immediately see that the situation in which both players are uninformed is strongly preferred by both of them to all other three situations.

Example 2.

$$G_A = \begin{pmatrix} 2,2 & 5,0 \\ 0,5 & 0,0 \end{pmatrix}, \qquad G_B = \begin{pmatrix} 0,0 & 0,15 \\ 15,0 & 6,6 \end{pmatrix}.$$

The I-U matrix is

	2-Inf	2-Uninf
1-Inf	$4,4$	$5.5,3$
1-Uninf	$3,5.5$	$3,3$

Information helps whoever has it. It helps more if the one who has it is the only one to have it.

Example 3.

$$G_A = \begin{pmatrix} 2,2 & 5,1 \\ 1,5 & 0,0 \end{pmatrix}, \qquad G_B = \begin{pmatrix} 0,0 & 0,15 \\ 15,0 & 6,6 \end{pmatrix}.$$

The I-U matrix is

	2-Inf	2-Uninf
1-Inf	4, 4	5.5, 3.5
1-Uninf	3.5, 5.5	3, 3

Information helps whoever has it, but now it helps even the one who doesn't have it.

Example 4.

$$G_A = \begin{pmatrix} 5,5 & 10,-6 \\ -6,10 & -5,-5 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -5,-5 & -12,8 \\ 8,-12 & 5,5 \end{pmatrix}.$$

The I-U matrix is

	2-Inf	2-Uninf
1-Inf	5, 5	7.5, −0.5
1-Uninf	−0.5, 7.5	0, 0

Information helps whoever has it. If one has it alone, she has a greater advantage and it is harmful to the one who doesn't have it.

Example 5.

$$G_A = \begin{pmatrix} 2,2 & -1,-6 \\ -6,-1 & -2,-2 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -20,-20 & -5,-5 \\ -5,-5 & 2,2 \end{pmatrix}.$$

The I-U matrix is

	2-Inf	2-Uninf
1-Inf	2, 2	−1.5, −1.5
1-Uninf	−1.5, −1.5	0, 0

Information is good for both, if both have it, bad for both, if only one has it. Put differently, the information of the two players is *complementary* to each other. The reason for this complementarity, as can be seen from the matrices, is that in order to take advantage of the knowledge about the state of nature, they have to coordinate, and to do that they both have to know the state.

Example 6.

$$G_A = \begin{pmatrix} 0,0 & 1,-1 & -1,10 \\ -2,-2 & -2,-2 & -3,-12 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -2,-1 & -1,1 & -2,-11 \\ -1,0 & 1,0.5 & -1,10 \end{pmatrix}.$$

Again the outcome of the average game (when no player is informed) is $(0,0)$ (by dominant strategies). If player I knows the state-game, and player II knows that player I knows the state-game, then, by strong domination, player II will face a situation of the following payoffs, depending on his moves:

$$\frac{1}{2}\begin{pmatrix} 0,0 & 1,-1 & -1,10 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -1,0 & 1,0.5 & -1,10 \end{pmatrix} = \begin{pmatrix} -0.5,0 & 1,-0.25 & -1,10 \end{pmatrix};$$

hence $(-1,10)$ is the outcome of the game.

The I-U matrix will be of the form:

	2-Inf	2-Uninf
1-Inf	*	$-1,10$
1-Uninf	*	$0,0$

That is, if player I is informed, and player II knows it, it is great for player II and bad for player I. Notice that, in this specific example, if it is common knowledge that I knows the state-game, it is irrelevant whether II knows it as well or not.

3 Zero-sum games

Consider a two-person zero-sum game. If the players share the same beliefs, then at least one of them will benefit from information and ask for it (if this is free). To see this statement formally consider the following game. Nature chooses one of two zero-sum state-games having the same sets of possible pure strategies for each of the two players. G_A is selected with probability p and G_B is selected with probability $(1-p)$. G_A, G_B and p are common knowledge. The state-game chosen by nature is not revealed to the players, unless at least one of them asks for this information. If this happens, then the state-game is revealed to both of them. We will assume that the two players decide simultaneously whether to ask for information. After this happens, the players play the revealed state-game, if it was revealed, or expect a payoff equivalent to playing the game $pG_A + (1-p)G_B$, if it was not revealed. Call v_A the value of G_A, v_B the value of G_B, and v_p the value of $pG_A + (1-p)G_B$. If $v_p > pv_A + (1-p)v_B$, then player I will not ask for information, but player II will. Vice versa, if $v_p < pv_A + (1-p)v_B$, player I will ask for information. In case of equality both players will be indifferent between asking and not asking.

We shall see now that phenomena of information refusal can arise also in zero-sum-games when we depart from the assumption that the players share the same beliefs.

Example 7. Let a game have a structure analogous to the one studied above, except for the fact that the players have different beliefs about nature's move. Player I thinks that nature chooses G_A with probability p (and G_B with probability $(1-p)$), while player II thinks that nature chooses G_A with probability q (and G_B with probability $(1-q)$). G_A, G_B, p, q are common knowledge. Let

$$G_A = \begin{pmatrix} -1 & 0 \\ 3 & 0 \end{pmatrix}, \qquad G_B = \begin{pmatrix} 0 & 0 \\ -1 & 3 \end{pmatrix}.$$

$p = 0.9$, $q = 0.1$. Then $v_A = v_B = 0$. If both players reject information, they will end up playing a game which is equivalent to the non-zero-sum game

$$\begin{pmatrix} -0.9, 0.1 & 0, 0 \\ 2.6, 0.6 & 0.3, -2.7 \end{pmatrix}.$$

In this game Bottom and Left are (strictly) dominant strategies. The payoff of playing (Bottom, Left) is $(2.6, 0.6)$, which is the (subjectively expected) outcome of the game and it is strictly better for both players than what they would expect by having the game revealed, namely $(0, 0)$.

In the above example the beliefs of the players were not only different, they were also *inconsistent in Harsanyi's sense*: There exists no common prior that could generate those beliefs as conditional distributions given different sets of private information.

Thus, we see that phenomena of information rejection do occur when the beliefs are inconsistent, whereas they can't occur in the case of common beliefs. In between these two models, we have the case when the beliefs are different but consistent. One can't hope for a generalization to this case of the positive result holding for common beliefs. In fact, such a result would state that in every equilibrium in such situations, players would ask for information with probability one. However, this is not true: There may be an equilibrium in which with positive probability both players reject information, as the following counterexample shows.

Example 8. As before nature chooses G_A or G_B. Each player can be of two types. Player I's (II's) possible types are $\mathrm{I}_1, \mathrm{I}_2$ ($\mathrm{II}_1, \mathrm{II}_2$). If both players are of even type, or if they are both of odd type, then the state-game played is G_A. If the parities of their types differ, then G_B is played. Each player gets to know his (her) own type, but not the type of his (her) opponent.

	II_1	II_2
I_1	G_A	G_B
I_2	G_B	G_A

Since we assume the existence of a common prior, the subjective probabilities of the players can be seen as conditional distributions, given the types, derived from the following common prior on the pairs of types of the players.

	II_1	II_2
I_1	pq	$p(1-q)$
I_2	$(1-p)q$	$(1-p)(1-q)$

Let the two state-games have the following payoff matrices

$$G_A = \begin{pmatrix} 2 & 0 \\ -1 & 0 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}.$$

As before, after nature chooses the game to be played, and each player is informed of his (her) type, each player decides whether to ask for information. If either one of the players asks for information, then the game is revealed to both of them. Otherwise they will make their move without additional information. In such a case, if player I is of type I_1, she will act as if the game $qG_A + (1-q)G_B$ were played. If she is of type I_2, she will act as if $(1-q)G_A + qG_B$ were played. Analogously, if player II is of type II_1, he will act as if $pG_A + (1-p)G_B$ were played; if he is of type II_2, he will act as if $(1-p)G_A + pG_B$ were played.

Considering the equilibria of this game, with the option of asking for public revelation, we have the following observations:

(a) Asking for information guarantees zero to both players since the values of G_A and G_B are zero. Notice that, even though in each of the state-games optimal strategies are not unique (for instance, in G_A, player II must play Right while I must play his Top strategy with probability of at least 1/3), there exists nevertheless a clear outcome of the game, namely zero.

(b) If $q = p = 1/2$, there is an equilibrium in which both players do not ask for revelation, independently of their types. This is so since if, say, player II does not ask for information, then player I, of both types, by not asking for information and playing Top, will get either 0 (if Left is played) or 1/2 (if Right is played). Similarly, if player I does not ask for information, it is an optimal reply for player II not to ask for it either. Note however that although not asking for revelation yields an equilibrium, it is still true that each player is indifferent between asking and not asking for information, in accordance with what was stated in the case of common beliefs (since $v_{1/2} = v_A = v_B = 0$).

Now we shall show that:

(c) For all values of q and p, there is a mixed strategy equilibrium that involves, with positive probability, the rejection of information by both players.

Given that $v_A = v_B = 0$, we can simplify the game and assume that, whenever at least one player asks for information, the payoff for both players will be zero. The set of possible pure strategies available to each type of player I (the row player) is N_T (i.e. reject information and play top if the game is not revealed), N_B (reject information and play bottom if the game is not revealed), and Y (ask for information, and play optimally afterwards). Analogously the set of pure strategies for each type of player II is (with the obvious meaning of the symbols) N_L, N_R, Y. Let (Y, N_R) be the strategy according to which player II plays Y if he is of type II_1 and N_R if he is of type II_2, and so on.

Let the row player play

(Y, N_T) with probability p
(N_T, Y) with probability $1-p$,

and the column player play

(Y, N_L) with probability q
(N_L, Y) with probability $1-q$,

To see that this is a Nash equilibrium (whose payoff is zero), observe that given that the column player is following this strategy and given that the row player is of type I_1, then, the conditional probability that the game is G_A, given that it is not revealed, is 1/2. Therefore player I_1 is indifferent between revealing the game or not revealing it and playing Top. Analogous considerations hold with the roles of the players reversed.

Under this Nash equilibrium with probability $4pq(1-p)(1-q)$ both players will reject free information. This probability of refusing information is positive except when $q \in \{0,1\}$ or $p \in \{0,1\}$, namely if (at least) one of the players knows the state-game.

4 Robustness results

In this section we will provide two examples in which we show that even a minor perturbation of the zero-sum-same-beliefs nature of a game can lead to information rejection.

In the first example, the two players share the same beliefs but the zero-sum structure of the game is perturbed.

Example 9. Nature chooses one of the following two state-games with equal probabilities.

$$G_A = \begin{pmatrix} 0,0 & 2,-2 \\ -2,2 & -\eta,-\eta \end{pmatrix}, \qquad G_B = \begin{pmatrix} 2\epsilon, 2\epsilon & -2,2 \\ 2,-2 & -\eta,-\eta \end{pmatrix}.$$

If the two players are not informed about the choice of nature, then they play the game

$$G_{1/2} = \begin{pmatrix} \epsilon,\epsilon & 0,0 \\ 0,0 & -\eta,-\eta \end{pmatrix}.$$

For $0 < \epsilon, \eta < 1$ the outcome of $G_{1/2}$ is (ϵ, ϵ), corresponding to (Top, Left), the outcome of G_A is $(0,0)$, corresponding to (Top, Left) and the outcome of G_B is $(-\eta, -\eta)$ corresponding to (Bottom, Right). Therefore both players strictly prefer not to have the game revealed whenever $0 < \epsilon, \eta < 1$. As $\epsilon \to 0$, and $\eta \to 0$, the game tends to a zero-sum-game, and for $\epsilon = \eta = 0$ both players are indifferent between having the game revealed or not. This shows that even a minor perturbation of a zero-sum game can produce the phenomenon of information rejection. Mathematically, this is the well known phenomenon of the discontinuity of Nash Equilibria as set-valued functions of the payoffs.

We will show now that even a slight change of the beliefs, that makes them inconsistent, can induce rejection of information in a zero-sum game.

Example 10. Consider now the following zero-sum game. Nature chooses one of the following two zero-sum state-games ($0 < \epsilon < 1/2$ is a fixed constant).

$$G_A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad G_B = \begin{pmatrix} -1 & 2\epsilon \\ 0 & \epsilon \end{pmatrix}.$$

It is common knowledge that player I thinks that nature has chosen G_A with probability p, whereas player II thinks that nature has chosen G_A with probability q. If they are not informed about nature's choice, they will play the non-zero-sum game

$$G_{p,q} = \begin{pmatrix} 2p-1, 1-2q & 2(1-p)\epsilon, -2(1-q)\epsilon \\ 0,0 & (1-p)\epsilon, -(1-q)\epsilon \end{pmatrix}.$$

If $p > 1/2$, and $q < 1/2$, then (Top, Left) is the outcome of the game $G_{p,q}$ (i.e. a unique strongly dominant strategies Nash Equilibrium), and its associated payoff is $(2p-1, 1-2q)$.

The Nash equilibria payoffs of G_A, G_B were all $(0,0)$. Therefore, for $p > 1/2$, and $q < 1/2$ there is strict preference for not receiving information. As $p \searrow 1/2$, and $q \nearrow 1/2$, the beliefs tend to be consistent, and when $p = q = 1/2$, the players are indifferent between having the game revealed or not.

This example is of the same kind as Example 7, except that here the phenomenon of information rejection holds whenever $p > 1/2$ and $q < 1/2$.

5 Coordination games

In this section we will consider a class of models in which nature chooses a game with a probability Θ, which is itself a random variable. First we will show that in coordination games of this type, it is always better (for each of the players) to have Θ revealed to both players rather than to receive any other signal (even private).

Let

$$G_A = \begin{pmatrix} \alpha,\alpha & 0,0 \\ 0,0 & \beta,\beta \end{pmatrix}, \qquad G_B = \begin{pmatrix} \gamma,\gamma & 0,0 \\ 0,0 & \delta,\delta \end{pmatrix},$$

with $0 \leq \beta < \alpha$ and $0 \leq \gamma < \delta$. Let Θ be a $[0,1]$-valued random variable such that, given $\Theta = \theta$, nature chooses game G_A with probability θ and G_B with probability $1-\theta$. Let Z be any random variable, and let the law of (Z,Θ) be common knowledge. We consider two variants of the game: one in which the value of Θ is revealed directly to both players, and one in which only the value of Z is revealed. The following theorem supports the intuition that the players are better off (they can better coordinate) when they publicly know the value of Θ rather than just the public signal Z associated with it.

Theorem 11. *When only (measurable) pure strategies are considered, the maximum expected payoff in equilibrium when the value of Θ is revealed to both players is greater or equal than the maximum expected payoff in equilibrium when only the value of Z is revealed to both players (but not that of Θ).*

Proof. Let Z be revealed. The set of (measurable) pure strategies of player I can be indexed by the Borel subsets of $[0,1]$. For $S \in \mathcal{B}([0,1])$, we will say that player I plays strategy σ^I_S, or simply that she plays S, if she plays Top when $E[\Theta|Z] \in S$ and Bottom when $E[\Theta|Z] \notin S$. Similarly, we say that player II plays $\widetilde{S}$ if he chooses Left or Right according to whether $E[\Theta|Z]$ is or is not in $\widetilde{S}$.

Write $M = E[\Theta|Z]$. Assume for simplicity that M is absolutely continuous with density f_M.

After seeing Z, each player attaches probability $E[\Theta|Z]$ to the fact that nature chooses G_A. Hence, the expected payoff when player I plays S, and player II plays $\widetilde{S}$ is

$$\int [\mathbf{1}_{S\cap\widetilde{S}}(m)(\alpha m + \gamma(1-m)) + \mathbf{1}_{S^c\cap\widetilde{S}^c}(m)(\beta m + \delta(1-m))] f_M(m)\ dm.$$

It is clear that a necessary and sufficient condition for a pure strategy equilibrium is that $\widetilde{S} = S$. In this case the payoff is

$$\int [\mathbf{1}_S(m)(\alpha m + \gamma(1-m)) + \mathbf{1}_{S^c}(m)(\beta m + \delta(1-m))] f_M(m)\ dm.$$

To determine a set S^* that maximizes the above payoff observe that

$$\alpha m + \gamma(1-m) \geq \beta m + \delta(1-m) \iff m \geq m^* := \frac{\delta - \gamma}{(\delta-\gamma)+(\alpha-\beta)}.$$

Notice that the assumptions on $\alpha, \beta, \gamma, \delta$ guarantee that $m^* \in (0,1)$. Thus, $S^* = [m^*, 1]$, and the maximum expected payoff in equilibrium is

$$\int \phi(m) f_M(m)\ dm = E[\phi(M)],$$

where $\phi : [0,1] \to \mathbb{R}$ is given by

$$\phi(m) = [\delta + (\beta - \delta)m]\mathbf{1}_{[0,m^*)}(m) + [\gamma + (\alpha-\gamma)m]\mathbf{1}_{[m^*,1]}(m).$$

The function ϕ is piecewise linear; it is easy to check that it is continuous (also at m^*). Furthermore, since $\beta - \alpha < 0 < \delta - \gamma$, we have that $\beta - \delta < \alpha - \gamma$. Hence ϕ is convex.

By Jensen's inequality, we have

$$E[\phi(M)] = E[\phi(E[\Theta|Z])] \leq E[E[\phi(\Theta)|Z]] = E[\phi(\Theta)].$$

The quantity $E[\phi(\Theta)]$ is the expected payoff when $Z = \Theta$ is shown and both players play S^*. □

Next, Let Z_{I} and Z_{II} be two real valued random variables related to the variable Θ through the joint distributions of (Θ, Z_{I}) and $(\Theta, Z_{\mathrm{II}})$. Consider the following three variants of the game: In all three games, nature chooses G_A with probability θ and G_B with probability $1-\theta$, where θ is the realization of the random variable Θ.

- In game Γ_0, each player i observes privately the signal Z_i (which can thus be thought of as his *type*).
- In game Γ_1, the value of Z_{I} is made public, and the players receive no additional private signals.
- In game Γ_2, the value of Z_{II} is made public, and the players receive no additional private signals.

The following theorem states that, in order to better coordinate, each of the players prefers that his (her) signal be made public rather than kept private.

Theorem 12. *The games $\Gamma_0, \Gamma_1, \Gamma_2$ have pure-strategy equilibria. Furthermore, let $u_i^j(z_i)$ ($i \in \{\mathrm{I}, \mathrm{II}\}$, $j \in \{0,1,2\}$) be the maximum of the pure-strategy equilibrium payoffs for player i in game Γ_j, when (s)he observed $Z_i = z_i$. Then*

$$u_{\mathrm{I}}^0(z_{\mathrm{I}}) \le u_{\mathrm{I}}^1(z_{\mathrm{I}}),\ \forall z_{\mathrm{I}} \in \mathbb{R},$$
$$u_{\mathrm{II}}^0(z_{\mathrm{II}}) \le u_{\mathrm{II}}^2(z_{\mathrm{II}}),\ \forall z_{\mathrm{II}} \in \mathbb{R}.$$

Proof. In $\Gamma_0, \Gamma_1, \Gamma_2$, (Top, Left) and (Bottom, Right) are pure strategy Nash equilibria in each of the three games.

Consider Γ_0. As seen in the proof of Theorem 11, a (measurable) pure strategy for player I is "play Top iff $z_{\mathrm{I}} \in S$," with $S \in \mathrm{Bor}(\mathbb{R})$. Analogously, a pure strategy for player II is of the form "play Top iff $z_{\mathrm{II}} \in D$". Suppose that S and D are such that an equilibrium is achieved when I plays S and II plays D. The payoff for player I is

$$\begin{aligned}
u_{\mathrm{I}}^0(z_{\mathrm{I}}) = \int & \mathbf{1}_S(z_{\mathrm{I}})\mathbf{1}_D(z_{\mathrm{II}})\big[\alpha E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}, Z_{\mathrm{II}} = z_{\mathrm{II}}] \\
& + \gamma E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}} Z_{\mathrm{II}} = z_{\mathrm{II}}]\big]\ dF_{Z_{\mathrm{II}}|Z_{\mathrm{I}}}(z_{\mathrm{II}}|z_{\mathrm{I}}) \\
& + \int \mathbf{1}_{S^c}(z_{\mathrm{I}})\mathbf{1}_{D^c}(z_{\mathrm{II}})\big[\beta E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}, Z_{\mathrm{II}} = z_{\mathrm{II}}] \\
& + \delta E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}} Z_{\mathrm{II}} = z_{\mathrm{II}}]\big]\ dF_{Z_{\mathrm{II}}|Z_{\mathrm{I}}}(z_{\mathrm{II}}|z_{\mathrm{I}}) \\
= \mathbf{1}_S(z_{\mathrm{I}}) & \int_D \big[\alpha E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}, Z_{\mathrm{II}} = z_{\mathrm{II}}] \\
& + \gamma E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}} Z_{\mathrm{II}} = z_{\mathrm{II}}]\big]\ dF_{Z_{\mathrm{II}}|Z_{\mathrm{I}}}(z_{\mathrm{II}}|z_{\mathrm{I}}) \\
& + \mathbf{1}_{S^c}(z_{\mathrm{I}}) \int_{D^c} \big[\beta E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}, Z_{\mathrm{II}} = z_{\mathrm{II}}] \\
& + \delta E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}} Z_{\mathrm{II}} = z_{\mathrm{II}}]\big] dF_{Z_{\mathrm{II}}|Z_{\mathrm{I}}}(z_{\mathrm{II}}|z_{\mathrm{I}}).
\end{aligned}$$

We can majorize the integral over D with the integral over $[0,1]$, and also the integral over D^c with the integral over $[0,1]$. Hence

$$\begin{aligned} u_{\mathrm{I}}^0(z_{\mathrm{I}}) \leq \mathbf{1}_S(z_{\mathrm{I}})\big[\alpha E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}] + \gamma E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}]\big] \\ + \mathbf{1}_{S^c}(z_{\mathrm{I}})\big[\beta E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}] + \delta E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}]\big]. \end{aligned}$$

We can improve the payoff by replacing S by the set that achieves the maximum equilibrium payoff in game Γ_1, say C:

$$\begin{aligned} u_{\mathrm{I}}^0(z_{\mathrm{I}}) &\leq \mathbf{1}_C(z_{\mathrm{I}})\big[\alpha E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}] + \gamma E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}]\big] \\ &\quad + \mathbf{1}_{(C)^c}(z_{\mathrm{I}})\big[\beta E[\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}] + \delta E[1-\Theta|Z_{\mathrm{I}} = z_{\mathrm{I}}]\big] \\ &= u_{\mathrm{I}}^1(z_{\mathrm{I}}) \\ &= u_{\mathrm{II}}^1(z_{\mathrm{I}}). \end{aligned}$$

Similarly, we prove that

$$u_{\mathrm{II}}^0(z_{\mathrm{II}}) \leq u_{\mathrm{I}}^2(z_{\mathrm{II}}) = u_{\mathrm{II}}^2(z_{\mathrm{II}}).$$

□

Corollary 13. *If (Θ, Z_{I}) and $(\Theta, Z_{\mathrm{II}})$ have the same law, then the sets that achieve the maximum equilibrium payoff in games Γ_1 and Γ_2 coincide, and*

$$u_{\mathrm{I}}^1 = u_{\mathrm{II}}^1 = u_{\mathrm{I}}^2 = u_{\mathrm{II}}^2.$$

The above positive results still leave room to some counterexamples about the acquisition of information in coordination games. The following example deals with a coordination game in which information is to be released at the request of at least one of the players, but none of them will ask for it. This is somehow counterintuitive, since it would appear that the disclosure of more information should lead to a better coordination.

Example 14. A random variable Θ has a beta distribution: $\mathcal{L}(\Theta) = \mathrm{Beta}(\alpha, \beta)$. The random variables $X_{\mathrm{I}}, X_{\mathrm{II}}, X, Y$ are i.i.d. conditionally on Θ and such that

$$P(X = 1|\Theta) = 1 - P(X = 0|\Theta) = \Theta.$$

Nature chooses the values of $\Theta, X_{\mathrm{I}}, X_{\mathrm{II}}, X, Y$. The random variable X_I is revealed to I only, and X_{II} is revealed to II only. Therefore I can be of two types, I_0 or I_1, according to the observed value of X_{I}. The same applies to player II.

Player I can either choose immediately T or B, or ask that the value of the random variable X be disclosed to both players, and then choose T or B. The same applies to player II. The payoff matrices are determined by the value of Y (which is never observed by either player):

$$\begin{pmatrix} 0,0 & 0,0 \\ 0,0 & 1,1 \end{pmatrix}, \quad \text{if } Y \text{ is } 0,$$

$$\begin{pmatrix} 1,1 & 0,0 \\ 0,0 & 0,0 \end{pmatrix}, \quad \text{if } Y \text{ is } 1.$$

A strategy of a player of a certain type, say I_0, is specified by a string of 4 actions. The first is either G (go) or S (stop), according to whether he asks for the disclosure of X or not. The next three actions are either T or B, and are the actions chosen by I_0 in the following three events: all the active players choose S, at least one of the active players chooses G and $X = 0$, at least one of the active players chooses G and $X = 1$. Similarly, we specify the strategy of players I_1, II_0 and II_1. Therefore, a strategy profile is specified by a string of 16 actions.

We claim that the parameters α and β of the common prior can be taken in such a way so as to have an equilibrium in which all players choose S.

Specifically, if $\alpha = 2.5$ and $\beta = 1$, then

$$\overset{(I_0)}{STBT}\ \overset{(I_1)}{STTT}\ \overset{(II_0)}{SLRL}\ \overset{(II_1)}{SLLL} \tag{1}$$

yields a (subgame perfect) equilibrium.

To see this let us first compute the expected payoffs of the players in the proposed strategies. The expected payoff of I_0 is

$$P_{I_0}(Y = 1) = P(Y = 1 | X_I = 0) = \frac{\alpha}{\alpha + \beta + 1} = \frac{5}{9}.$$

The expected payoff of II_0 is the same.

Similarly the expected payoff of I_1 (and of II_1) is

$$P_{I_1}(Y = 1) = P(Y = 1 | X_I = 1) = \frac{\alpha + 1}{\alpha + \beta + 1} = \frac{7}{9}.$$

Now we will compute the payoffs when one of the players deviates. By $*$ we will indicate any action available to the player.

If player I plays $(SB**)$, she gets 0.

Let us compute now the expected payoff of I_0 if she plays $(G * BT)$

$$\begin{aligned} &P_{I_0}(X = 0, X_{II} = 0, Y = 0) + P_{I_0}(X = 1, Y = 1) \\ &= \frac{\beta + 1}{\alpha + \beta + 1}\frac{\beta + 2}{\alpha + \beta + 2}\frac{\beta + 3}{\alpha + \beta + 3} + \frac{\alpha}{\alpha + \beta + 1}\frac{\alpha + 1}{\alpha + \beta + 2} \\ &= \frac{647}{1287} < \frac{5}{9}. \end{aligned}$$

The payoff of I_0, when she plays $(G * TT)$ is

$$\begin{aligned} &P_{I_0}(X = 0, X_{II} = 1, Y = 1) + P_{I_0}(X = 1, Y = 1) \\ &= \frac{\beta + 1}{\alpha + \beta + 1}\frac{\alpha}{\alpha + \beta + 2}\frac{\alpha + 1}{\alpha + \beta + 3} + \frac{\alpha}{\alpha + \beta + 1}\frac{\alpha + 1}{\alpha + \beta + 2} \\ &< \frac{647}{1287} < \frac{5}{9}. \end{aligned}$$

Furthermore for I_0 $(G*BB)$ is dominated by $(G*BT)$ and $(G*TB)$ is dominated by $(G*TT)$. Hence I_0 has no incentive to deviate from (1).

Similarly, the expected payoff of I_1, if she plays $(G*TT)$, is

$$\begin{aligned} & P_{I_1}(X=1, X_{II}=0, Y=1) + P_{I_1}(X_{II}=1, Y=1) \\ = & \frac{\alpha+1}{\alpha+\beta+1}\frac{\beta}{\alpha+\beta+2}\frac{\alpha+2}{\alpha+\beta+3} + \frac{\alpha+1}{\alpha+\beta+1}\frac{\alpha+2}{\alpha+\beta+2} \\ = & \frac{105}{143} < \frac{7}{9}. \end{aligned}$$

If I_1 plays $(G*BT)$, she gets

$$\begin{aligned} & P_{I_1}(X=0, X_{II}=0, Y=0) + P_{I_1}(X_{II}=1, Y=1) \\ = & \frac{\beta}{\alpha+\beta+1}\frac{\beta+1}{\alpha+\beta+2}\frac{\beta+2}{\alpha+\beta+3} + \frac{\alpha+1}{\alpha+\beta+1}\frac{\alpha+2}{\alpha+\beta+2} \\ < & \frac{105}{143} < \frac{7}{9}. \end{aligned}$$

For I_1 $(G*TB)$ is dominated by $(G*TT)$, and $(G*BB)$ is dominated by $(G*BT)$.

The argument for player II is completely analogous. Thus, the profile (1) yields an equilibrium.

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