Frustration and Anger in Games

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Introduction

Frustration, anger, and aggression have important consequences for economic and social behavior.

- Anger arises from the frustration of non-attainment of an expected outcome; as a behavioral consequence, this goal-blockage experience can lead to aggressive behavior.
- Emotions depend on beliefs and then we need belief-dependent preferences to illustrate anger-like motivations.
- We develop a formal framework and a set of models that incorporate frustration and anger in games.

Motivation & Examples

The following is inconsistent with standard social preferences (e.g., inequity aversion), but consistent with our model(s):

- **Psychology**: frustration-aggression hypothesis (Dollard et al., 1939). "Experiences of anger consist of the experience of an event as obstructing one's goals and as caused by someone else's blameworthy intent." (Frijda, 1993).
- **Facts (empirics):**
  - unexpected losses by home football/soccer teams are associated with increased domestic violence (Card & Dahl, 2011) or violent crime (Munyo & Rossi 2013).
  - firms do not want to "antagonize customers" (Anderson, E. and D. Simester 2010)
- **Facts (experimental):**
  - Self-reported anger correlates with *punishment* of free-riders in *Public Good Games* (Fehr & Gächter, 2002)
  - Rejections in the *Ultimatum Game* correlate with (manipulated) initially expected offers (Sanfey, 2009; Xiang *et al.*, 2013, with fMRI)
  - Deviations from expectations drive both anger and the destruction of endowments in *Power-to-Take Games* (van Winden *et al.* 2002,'05).
  - *Agents are blamed by principals* for bad outcomes (Gurdal *et al.* 2014).

Setting

Consider **multistage games with observable actions**:

- Players $i \in I$
- Histories/nodes h: $\varnothing, a^1, (a^1, a^2), ... \in \bar{H}$ ($\varnothing$=root of tree $\bar{H}$)
- Paths, or terminal nodes $z = (a^1, ..., a^T) \in Z$, where $a^t = \left(a_i^t\right)_{i \in I}$ is a profile of actions
- Monetary payoffs $\pi(z) = (\pi_i(z))_{i \in I}$
- Beliefs about paths conditional on each nonterminal history $h \in H := \bar{H} \backslash Z$:
  - **First-order beliefs**: $\alpha_i = (\alpha_i(\cdot|h))_{h \in H} \in \Delta_i^H(Z)$
  - **Second-order beliefs**: $\beta_i = (\beta_i(\cdot|h))_{h \in H} \in \Delta_i^H\left(Z \times \Delta_{-i}^H(Z)\right)$
  - **Coherence** of (α_i, β_i): $\alpha_i(\cdot|h) = \mathrm{marg}_Z \beta_i(\cdot|h)$ for each $h \in H$
- Main focus: 2-*stage games* with *complete information* (simplicity)

Frustration

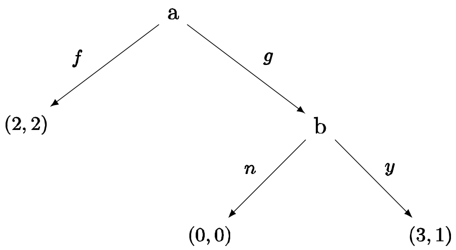
- Anger is anchored in frustration.
- **Frustration** is given by *unavoidable shortfall* in expected material payoff; thus, it depends on his beliefs about others and on own plans.

Player i's frustration, in the second stage given a^1, is defined as:

$$\mathrm{F}_i(a^1;\alpha_i) = \left[\mathbb{E}[\pi_i;\alpha_i] - \max_{a_i^2 \in A_i(a^1)} \mathbb{E}[\pi_i|(a^1,a_i);\alpha_i]\right]^+$$

where $[x]^+ = \max\{0,x\}$.

Example: Ultimatum mini-Game



If Bob initially expects f (fair offer), his frustration following g (greedy o.) is

$$\mathrm{F}_b(g;\alpha_b) = [2 \cdot (1 - \alpha_b(g)) + 1 \cdot \alpha_b(g)\alpha_b(y|g) - 1]^+ .$$

Given g, Bob is more frustrated (i) the more he expects the fair offer f, and (ii) the less he initially plans to reject the greedy offer g.

- *How do players react to frustration?*

Blame

A frustrated player may go after his co-players depending on the evaluation of how much his opponents can be blamed for the outcome which frustrates him.

- B_{ij} measures the amount of frustration i attributes to j, with:

$$\mathrm{B}_{ij}(a^1;\beta_i) \leq \mathrm{F}_i(a^1;\alpha_i)$$

- Player i moving at the second stage chooses a_i to maximize his "decision utility" of the form

$$u_i(a^1, a_i;\beta_i) = \pi_i(a^1, a_i) - \theta_i \sum_{j \neq i} \mathrm{B}_{ij}(a^1;\beta_i)\pi_j(a^1, a_i)$$

  where $\theta_i \geq 0$ is a sensitivity parameter.
- Three approaches to incorporate anger into utility functions according to different levels of cognitive appraisal, reflected by different blame functions.

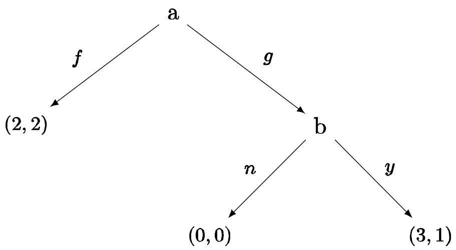
Simple Anger (SA)

A player's tendency to hurt others is proportional to his frustration, un-modulated by cognitive appraisal of blame:

$$\mathrm{B}_{ij}(a^1; \alpha_i) = \mathrm{F}_i(a^1; \alpha_i)$$

- *Frustration-aggression displacement hyphothesis* (Dollard *et al.*, 1939): the existence of frustration leads to some form of aggressive behavior through a displacement effect that directs hostile inclinations at substitute targets.
- *Card & Dahl, 2001*: correlation between an external source of frustration from unexpected loss by local teams and an increasing number of reports of domestic abuse.

Simple Anger: Ultimatum mini-Game



- If Bob initially expects f with certainty, his frustration following g is $\mathrm{F}_b(g; \alpha_b) = [2 \cdot 1 + 0 \cdot \alpha_b(y|g) - 1]^+ = 1$. Therefore $u_b^{SA}(g, n; \alpha_b) - u_b^{SA}(g, y; \alpha_b) = 3\theta_b - 1$. Bob rejects g if $\theta_b \geq 1/3$; otherwise, he accepts.
- If Bob initially expects g with certainty, his frustration following g is $\mathrm{F}_b(g; \alpha_b) = [2 \cdot 0 + 1 \cdot \alpha_b(y|g) - 1]^+ = 0$. Bob accepts g for every θ_b and $\alpha_b(y|g)$.

Anger from Blaming Behavior (ABB)

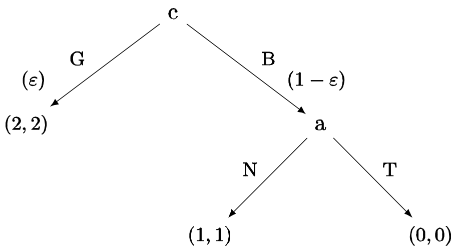
A player blames whoever causes his frustration.

How much i blames j is determined by a continuous function $\mathrm{B}_{ij}(a^1;\alpha_i)$ such that:

$$\mathrm{B}_{ij}(a^1;\alpha_i) = \begin{cases} 0, & \text{if } \{j\} \neq I(\varnothing) \\ \mathrm{F}_b(g;\alpha_b), & \text{if } \{j\} = I(\varnothing) \end{cases}$$

- In *leader-followers games*, SA and ABB are equivalent.
  For example, in the Ultimatum Minigame the two models yield to the same behavioral prediction.

Example: Hammering One's Thumb



c=chance, a=Andy, b=Bob (inactive, 2nd payoff). Assume that $\varepsilon < 1/2$. Following Bad day, we have:

$$\mathrm{F}_i(B;\alpha_a) = (1-\varepsilon)\cdot 2 + \varepsilon\alpha_a(N|B)\cdot 1 - 1 > 0$$

Difference between simple anger and anger from blaming behavior:

- **SA:** given B, Andy chooses T for θ_a sufficiently high;
- **ABB:** given B, Andy chooses N regardless of θ_a.

ABB: Could-Have-Been Blame

We propose two specific functional forms for ABB:

1. **Could-have-been blame**
   A frustrated player i considers for each co-player j what i would have obtained at most, in expectation, had j chosen differently:

$$\mathrm{B}_{ij}(a^1, \alpha_i) =$$

$$\min\left\{\left[\max_{a'_j \in A_j(\varnothing)} \mathbb{E}[\pi_i|(a^1_{-j}; a'_j); \alpha_i] - \mathbb{E}[\pi_i|a^1; \alpha_i]\right]^+ ; \mathrm{F}_i(a^1; \alpha_i)\right\}$$

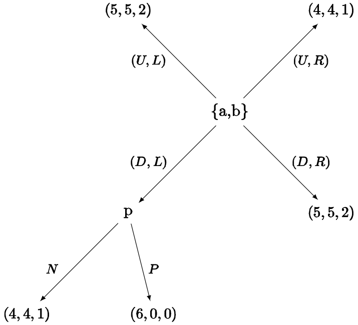
ABB: Blaming Unexpected Deviations

2 **Blaming unexpected deviations**
A frustrated i assesses for each co-player j how much i would have obtained had j behaved as expected:

$$\mathrm{B}_{ij}(a^1, \alpha_i) =$$

$$\min\left\{\left[\sum_{a'_j \in A_j(\varnothing)} \alpha_{ij}(a'_j)\mathbb{E}[\pi_i|(a^1_{-j}, a_j); \alpha_i] - \mathbb{E}[\pi_i|a^1; \alpha_i]\right]^+ ; \mathrm{F}_i(a^1; \alpha_i)\right\}$$

Could-Have-Been Blame: Asymmetric Punishment



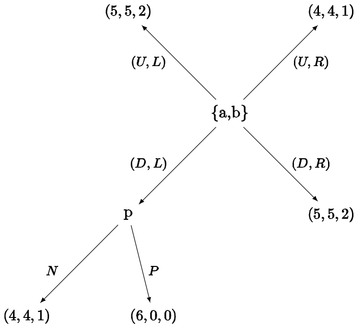
Consider Penny at $a^1 = (D, L)$.

$$\mathrm{B}_{pa}((D, L); \alpha_p) = \mathrm{B}_{pb}((D, L); \alpha_p) =$$

$$\min\{[2 - \mathbb{E}[\pi_p|(D, L); \alpha_p]]^+, [\mathbb{E}[\pi_p; \alpha_p] - 1]^+\} = [\mathbb{E}[\pi_p; \alpha_p - 1]]^+$$

- Penny fully blames both Ann and Bob for her frustration, but can and does punish only Bob.

Blaming Unexpected Deviations: Asymmetric Punishment



If Penny is initially certain of (U, L) then $\mathbb{E}[\pi_p; \alpha_p] = 2$. Given (D, L), her frustration is $\mathrm{F}_p((D, L); \alpha_p) = [2 - 1]^+ = 1$.

- Penny fully blames Ann, who deviated from U to D:

$$\mathrm{B}_{pa}((D, L); \alpha_p) = \min\left\{\left[2 - \mathbb{E}[\pi_p | a^1; \alpha_p]\right]^+; 1\right\} = 1$$

- Penny does not blame Bob, who played L as expected.

Anger from Blaming Intentions (ABI)

A player blames whoever he believes *intended* to cause his frustration.

- A frustrated player asks himself whether the co-player intended to give her a low expected payoff:

$$\mathrm{B}_{ij}(a^1;\beta_i) =$$

$$\min\left\{\mathbb{E}\left[\max_{a_j}\sum_{a^1_{-j}}\alpha_{j,-j}(a^1_{-j})\mathbb{E}[\pi_i|(a^1_j,a^1_{-j});\alpha_j]-\mathbb{E}[\pi_i;\alpha_j]\,\middle|\,a^1;\beta_i\right],\mathrm{F}_i(a^1,\alpha_i)\right\}$$

- Key issue: is observed behavior interpreted as *intentional*? According to the “trembling-hand” story, deviations from equilibrium are unintentional⇒no blame, no aggression!

Sequential Equilibrium: Consistent Assessments

Fix psychological utility function u_i ($u_i = u_i^{SA}, u_i^{ABB}, u_i^{ABI}$, or other).

An **assessment** is a profile of behavioral strategies and coherent beliefs $(\sigma_i, \alpha_i, \beta_i)_{i \in I}$ such that σ_i is the *plan* $\alpha_{i,i}$ entailed by second-order belief β_i.

An assessment $(\sigma_i, \beta_i)_{i \in I}$ is **consistent** if, for all i, h and $a = (a_j)_{j \in I}$,

(a) (behav.strat. given by 1^{st}-ord. beliefs) $\alpha_i(a|h) = \prod_{j \in I(h)} \sigma_j(a_j|h)$,

(b) (correct 2^{nd}-ord. beliefs) $\mathrm{marg}_{\Delta^H_{-i}(Z)} \beta_i(\cdot|h) = \delta_{\alpha_{-i}}$,

where α_i is derived from β_i (coherence) and $\delta_{\alpha_{-i}}$ is the Dirac probability measure that assignes probability 1 to the singleton $\{\alpha_{-i}\}$.

Sequential Equilibrium: Sequential Rationality

An assessment $(\sigma_i, \beta_i)_{i \in I}$ is a **sequential equilibrium (SE)** if

1. it is consistent and
2. it satisfies the following **sequential rationality** condition (one-shot-deviation property):

$$\forall i \in I, \forall h \in H, \mathrm{Supp}\sigma_i(\cdot|h) \subseteq \arg\max_{a_i \in A_i(h)} u_i(h, a_i; \beta_i).$$

Equilibrium Analysis

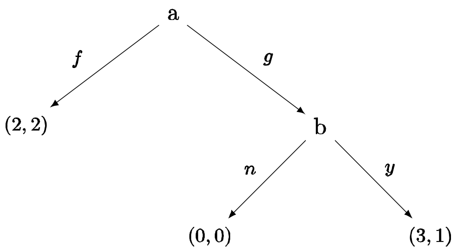
Theorem (cf. B&D, 2009)
If $u_i(h, a_i; \cdot)$ *is continuous for all* $i \in I, h \in H$ *and* $a_i \in A_i(h)$*, then there is at least one SE.*

- Every game with SA, ABB, or ABI has at least one SE.

Proposition
In every perfect-information *game form with* no chance *move and a* unique SE of the material-payoff game*, this unique material-payoff equilibrium is* realization-equivalent to an SE of the psychological game *with ABI, ABB, or – with only two players – SA.*

- *Intuition:* PI⇒*pure-strat.* material-payoff eq.⇒No on-path surprise⇒No on-path anger with psy-utility functions⇒On-path material payoff maximization and angry punishment of deviations makes deviations even less attractive.

Example: Sequential Equilibria of UmG



- Under all models (SA, ABB, ABI), (g, y) is SE for all θ_b (illustration of previous Proposition):
  - if greedy offer g is expected no anger$\Rightarrow$ *yes*
- Under **SA** and **ABI**, also (f, n) is SE if θ_b is high enough:
  - off-path offer g is unexpected$\Rightarrow$anger$\Rightarrow$ *no* for suff. high θ_b (credible threat)
- Under **ABI** (f, n) is *not* SE:
  - off-path offer g interpreted as *unintentional mistake* (trembling hand)$\Rightarrow$no blame$\Rightarrow$ n not credible threat

Discussion

- *Dynamic Inconsistency of preferences*: implied by F&A models, but we assume sophistication⇒no change of plan
- *T-period extension*: fast vs. slow play version
  - fast play: benchmark is initial expectation
  - slow play: benchmark is beginning-of-period expectation
  - cooling off effect (experiment by Gneezy and Imas, 2014)
- *Unintuive implication of SE:* deviations are not blamed and punished, because they are perceived as unintended
- Do not blame (sic) us, we are true to the original concept. Blame instead the trembling-hand notion of consistency built into (original) SE, and look for different solution concepts!
- *Polymorphic SE:* minimal generalization of SE allowing meaningful on-path updating about others' intentions (even with complete information!)

Discussion (cont.)

- We make sense of relevant evidence with psychological models where anger is belief-dependent: unavoidable shortfall
- Anger can also depend on regret (e.g., unexpected discounts after purchase, see Anderson & Simester, 2010), which is belief-dependent in a different way ...
- ... or on perceived unfairness (riots, political unrest), which—however—often is hard to disinguish from deviation from expectations
- *F&A* vs *Negative Reciprocity*: both give credibility to threats, but unlike NR, in the F&A models no aggression occurs on a pure-equilibrium path⇒no miserable equilibrium
- *F&A* vs *Guilt Aversion:* complementary "committment devices," GA (Battigalli & Dufwenberg 2007) supports on-path promises, F&A supports off-path threats

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